Performance measures of change point detection schemes in theory and application

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Outline

Notational Preliminaries

The change-point model

Sequence of rv $X_1, X_2, \ldots$ with cdf $\{F_{(i)}\}$ and a certain (unknown) time point m = **change-point** with

$$F_{(i)} = \begin{cases} F_0 & , \ i < m \\ F_1 & , \ i \geq m \end{cases} .$$

Example: $F_0 = \mathcal{N}(\mu_0, 1)\,,\ F_1 = \mathcal{N}(\mu_1, 1)$ + independence

Control charts

Different names, same concepts:
control charts, change point detection, continuous inspection, surveillance, monitoring, fault detection ...

Aim:
Detect rapidly and reliably, whether there appeared change-point m!

- Transformation $\{X_i\}_{i=1,2,\dots,n} \to Z_n$ and
- Stopping time $L = \min\{n \in \mathbb{N}: \ Z_n \ \notin \ \mathcal{O} = [c_l^*, c_u^*]\}$.

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Control chart performance measures

The dominator – Average Run Length (ARL)

Notation: $E_m(.)$ expectation for given change-point m.

Definition:

$$ARL = \begin{cases} E_\infty(L) & \text{, process in control} \\ E_1(L) & \text{, process out of control} \end{cases} .$$

Note that for dealing with the ARL, the sequence $\{X_i\}$ is (strong) stationary with the same probability law for all i. Thus, e. g., for any μ (and not only μ_0 and μ_1)

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Sort of history of performance measuring

1. SHEWHART (192X,193X) similar to tests: error probabilities,
2. AROIAN/LEVENE (1950) average spacing number and average efficiency number,
3. GIRSHICK/RUBIN (1952) Bayesian framework,
4. PAGE (1954) introduced term ARL *as the average number of articles inspected between two successive occasions when rectifying action is taken.*
5. BARNARD (1959) *If it were thought worthwile one could use methods analogous to these given by Page (1954) and estimate the average run length as a function of the departure from the target value. However, as I have already indicated, such computations could be regarded as having the function merely of avoiding unemployment amongst mathematicians.*

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History etc. II

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$$P(M=m) = \begin{cases} \pi & , \ m = 0 \\ (1-\pi)\,(1-p)^{m-1} p & , \ m > 0 \end{cases} \ , \ \pi \in [0,1)\,,\ p \in (0,1)$$

and minimize

$$\begin{cases} P_{\pi,p}(L < M) + c\,E_{\pi,p}(L-M)^+ & \text{for all s.t. } L \\ E_{\pi,p}(L-M \mid L \geq M) & \text{for all s.t. } L \text{ with } P_{\pi,p}(L<M) \leq \alpha \end{cases}$$

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History etc. III

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    - detection prob. $P(L = t \mid M \leq t)$ and false alarm prob. $P(L = t \mid M > t)$,
    - expected delay $ED(m) = E\big((L - m)^+ \mid M = m\big)$,
    - conditional expected delay $CED(m) = ED(m)/P(L \geq m)$,
    - summarized expected delay $ED = E\big((L - M)^+\big)$,
    - probability of successful detection $PSD(t, d) = P(L - M < d \mid L \geq M, M = m)$,
    - Predictive Value (of an alarm) $PV(t) = P(M \leq t \mid L = t)$.
11. BASSEVILLE/NIKIFOROV (1993)
    - mean time between false alarms $E_{\mu_0}(L)$ – ARL,
    - conditional mean delay $D^*_m = E_{\mu_1}\big(L - m + 1 \mid L \geq m, \mathcal{F}_{m-1}\big)$,
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History etc. IV

12. Lai (1995) *It is therefore much more relevant to consider*
    (a) *the probability of no false alarm during a typical (steady state) segment of the base-line period and*
    (b) *the expected delay in signaling a correct alarm,*

    instead of the ARL which is the mean duration to the first alarm assuming a constant in-control or out-of-control value.

Schemes under consideration

Collection of one-sided schemes

- CUSUM: Page (1954)

$$Z_n = \max\left\{0, Z_{n-1} + X_n - k\right\}, \; Z_0 = z_0 ,$$
$$L = \inf\left\{n \in \mathbb{N} : Z_n > h\right\} \qquad \left(k = (\mu_0 + \mu_1)/2\right)$$

- EWMA: Roberts (1959) (reflecting barrier – Waldmann (1986), Gan (1993))

$$Z_n = \max\left\{z^*_{\text{reflect}}, (1-\lambda)\, Z_{n-1} + \lambda\, X_n\right\}, \; Z_0 = z_0 ,$$
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- CUSUM is optimal in terms of LORDEN'S $\mathcal{W}$,
- (steady-state started) GRSR is asymptotically optimal for $\mathcal{D}_{\text{PS}}$ of POLLAK/SIEGMUND,
- Bayes/LR is optimal for the Bayesian designs,
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Calculation

Connections between the considered measures

- **Shewhart chart**

  $\mathcal{W} = \mathcal{D} = \mathcal{D}_{\mathrm{PS}} = \mathcal{L} = E_1(L) = E_m(L - m + 1 | L \geq m)$.

- **CUSUM**

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  modifications: $\mathcal{D} = \mathcal{D}_{\mathrm{PS}} \neq \mathcal{L}$.

But:

- **EWMA**

  All measures provide different values.

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Measures to be considered and calculated

- (zero-state) ARL $\mathcal{L}$,
- steady-state ARL $\mathcal{D}$,
- false alarm probability $P(L < M) = \sum_{m=1}^{\infty} P_m(L < m) P(M = m)$,
- expected delay

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Results – overview

- in-control ARL vs. false alarm probability α for $p \in \{0.1, 0.01, 0.001\}$,
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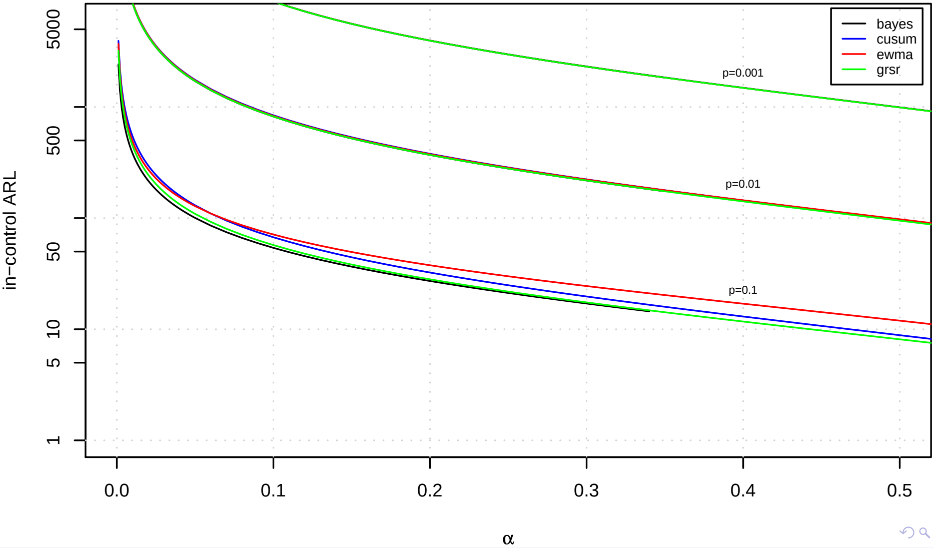
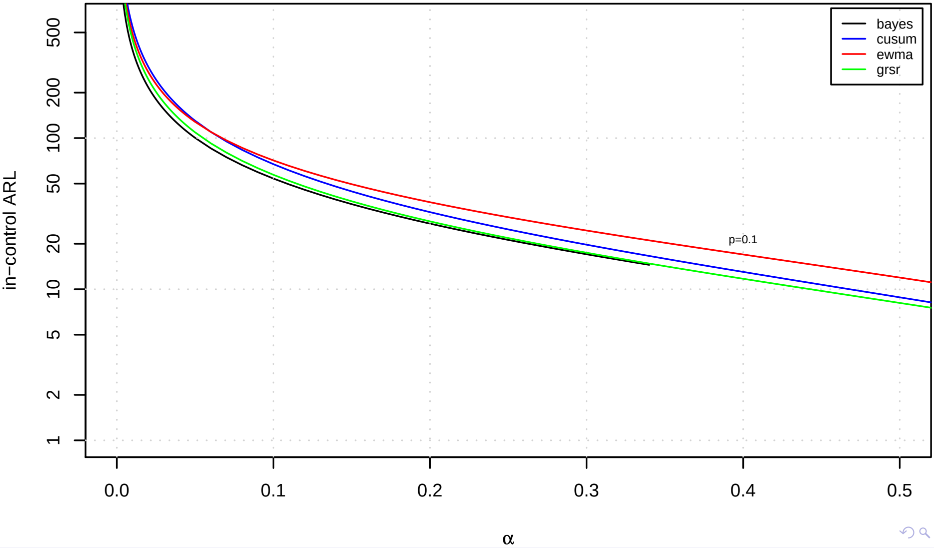


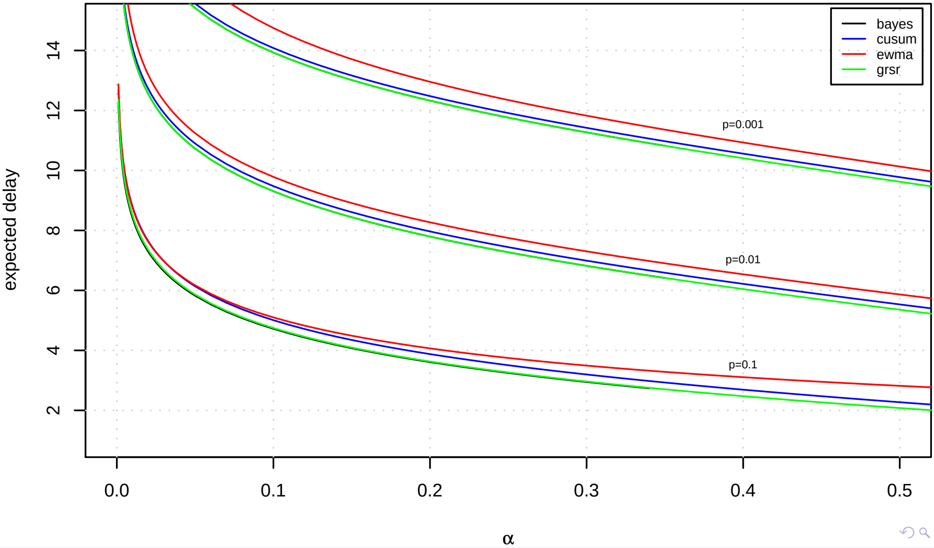
in-control ARL vs. false alarm probability α II



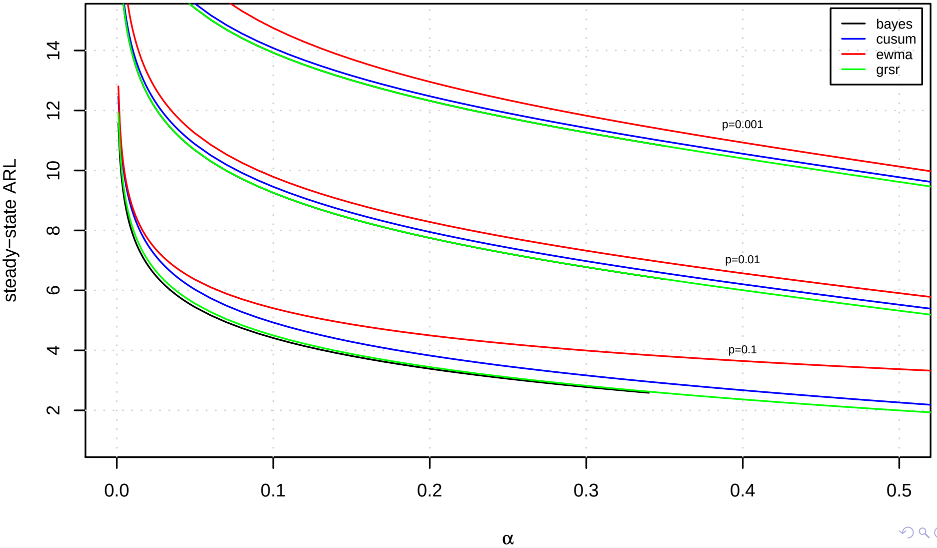
Remarks in-control ARL

- Only slight differences between the 4 schemes.
- They disappear completely for decreasing *p*.
- The Bayes scheme has the lowest values followed by GRSR, CUSUM, and EWMA.

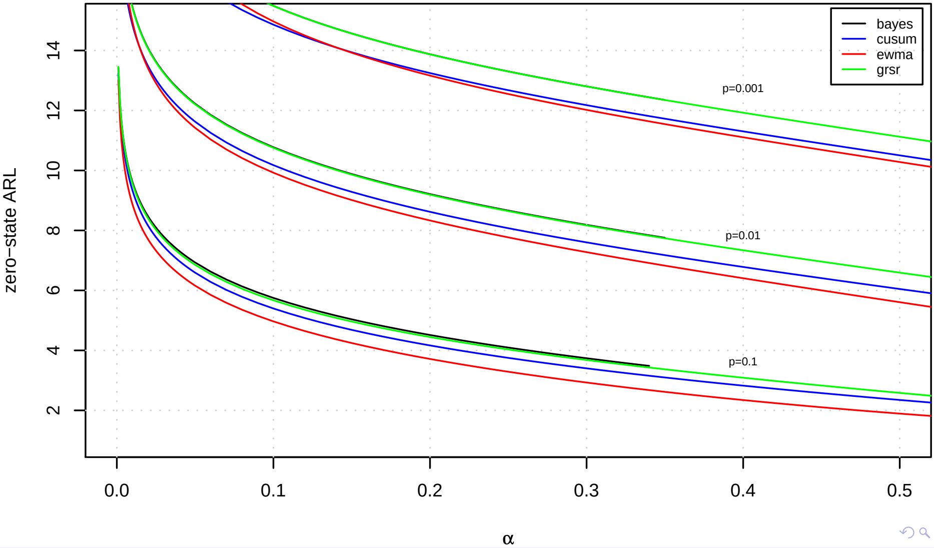
expected delay *ED* vs. false alarm probability α



steady-state ARL vs. false alarm probability α



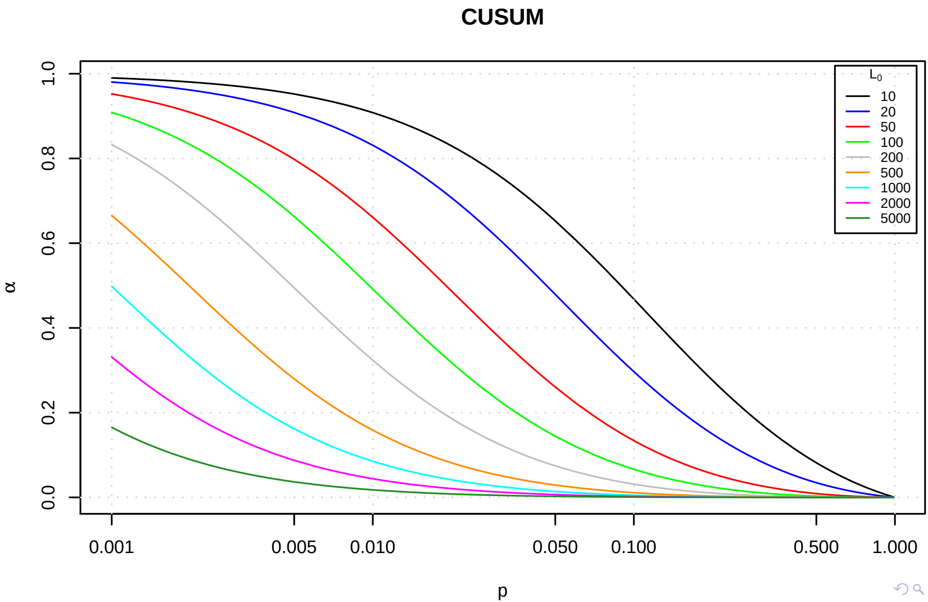
zero-state ARL vs. false alarm probability α



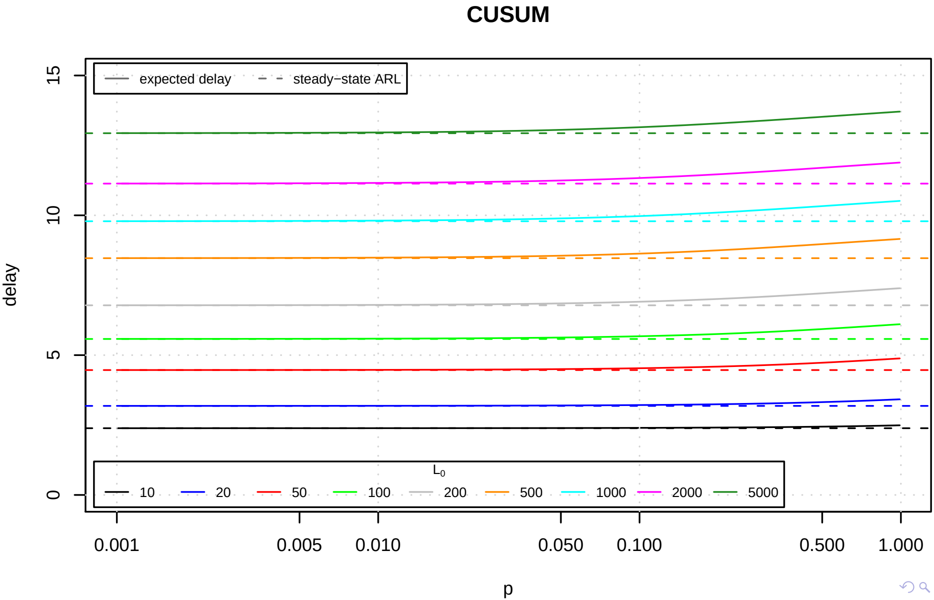
Remarks out-of-control ARL like measures

- Again, only slight differences between the 4 schemes.
- They become smaller for decreasing p except for EWMA in general and for CUSUM and EWMA for the zero-state ARL.
- For the expected delay ED and for the steady-state ARL, the Bayes scheme has the lowest values followed by GRSR, CUSUM, and EWMA.
- For the zero-state ARL, EWMA is the best followed by CUSUM, GRSR, and Bayes.
- For Bayes and GRSR, steady-state ARL is smaller than the ED, for EWMA is it vice versa, while for CUSUM it seems to be stable.

false alarm probability α vs. Bayes p for CUSUM only



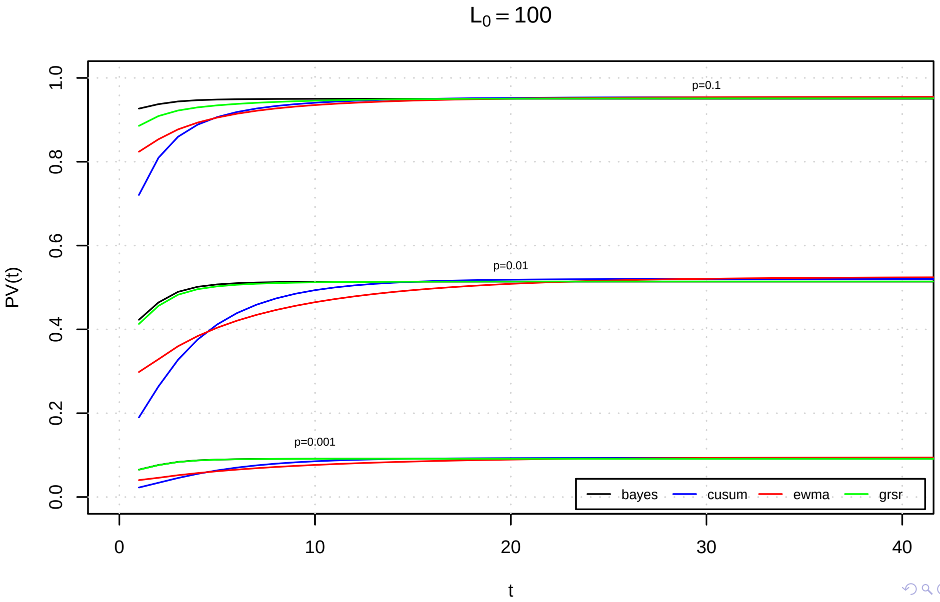
delays vs. Bayes p for CUSUM only



Remarks CUSUM with certain in-control ARL values

- For the considered in-control ARL values, many reasonable (p, α) configurations are possible.
- The delay (both the expected delay ED and the steady-state ARL) behaves nearly robust against varying p.

predictive value for some Bayes p



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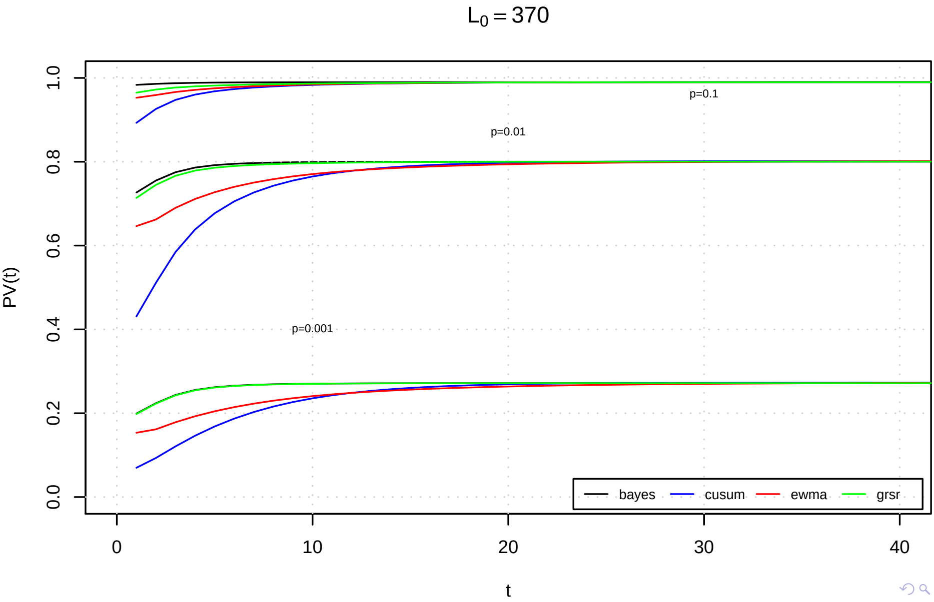

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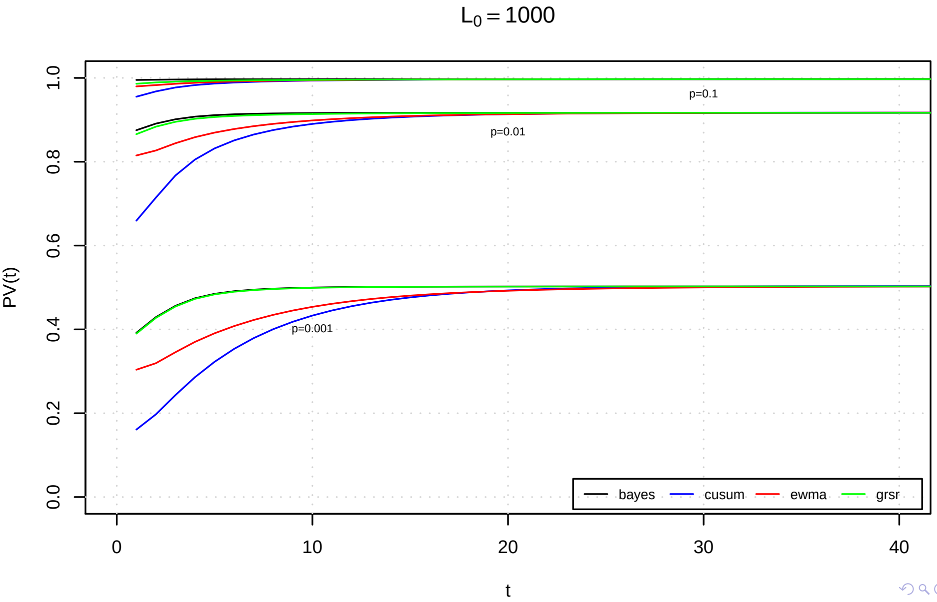

Some numbers explaining shape of previous $PV(t)$

$\mathcal{L}_0$	p	α CUSUM	EWMA	GRSR	Bayes
100	.1	.07	.07	.06	.05
	.01	.49	.49	.49	*.49*
	.001	.91	.91	.91	*.91*
370	.1	.02	.01	.01	.01
	.01	.20	.20	.20	.20
	.001	.73	.73	.73	*.73*
1000	.1	.00	.00	.00	.00
	.01	.08	.08	.08	.08
	.001	.50	.50	.50	*.50*

Remarks predictive value $PV(t)$

- Results are similar to FRISÉN/WESSMAN (1999), now also for larger in-control ARL values and smaller p.
- For the reasonable (p, L_0) configurations $((0.1, 100), (0.1, 1000), (0.01, 1000))$, Bayes and GRSR have nearly constant $PV(t)$ values, while EWMA and CUSUM have considerably decreased values for small t.
- The behavior of CUSUM is quite surprising, because it starts from worst-case (that is usually softened by a head-start).

In the light of daily practice

Daily Practice: Fast Conclusions

- All schemes are equal.
- Bayesian approach allows judgment of risk, but the related schemes do not offer "added value".
- Except the embedded robustness of CUSUM against inertia effects no further "unique selling point" could be seen.
- If one is beyond Shewhart charts (and Western Electric or other Runs Rules), then any scheme could be deployed (given that all the other possible trouble is addressed like correlated data, mixture data, wrongly picked models, for wrong out-of-control μ designed, etc.)

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- Software restrictions: AMTC's (WinSPC) and AMD's (ASPECT) SPC software packages offer only Shewhart charts and many different Runs Rules flavors. The Infineon/Qimonda package (SPACE) allows usage also of EWMA (and MA) charts. It seems so that this fits to the commercial SPC software market in general.
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- Development and tuning of the in-control models is very challenging. Most of the time is needed for picking the right parameters (a dry etch or an eBeam writing process provides thousands of time series on a 1-second time grid per job). One keyword in semiconductor industry (of course not only there) is FDC (fault detection and classification).
- Handy rules for setting up control chart design needed. See next slide for two-sided EWMA control chart setup. (quick and dirty $3\sqrt{(\lambda/(2-\lambda))}$ instead of deploying all those nice approaches such as Markov chain, integral equation etc.)

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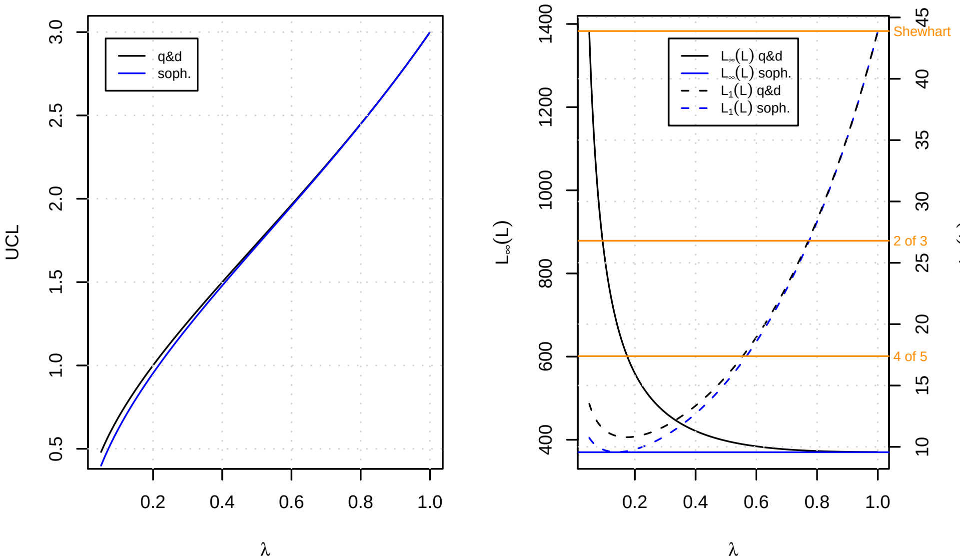
EWMA: Quick and dirty vs. sophisticated



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- Most of the performance measures of change-point detection schemes could be calculated accurately.
- The more sophisticated schemes/control charts exhibit similar properties.
- For practice, there is no clear favorite.
- Only one of the considered scheme (EWMA) is available in commercial SPC software packages (it is the worst among the considered 4).
- Current challenge of change-point detection in practice is the choice of a reasonable in-control model including a reliable understanding of “detectable” deviations (write down a suitable OCAP [out-of-control action plan]).

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