SPC in practice, control charts with runs rules, and EWMA

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Outline

1. Introduction
2. Runs Rules for control charts
3. Competitors and competition
4. Conclusions

Introduction

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- …
- **S**tatistical **P**rocess **C**ontrol.

SPC synonyms?!

- Change point detection.
- On-line monitoring.
- Surveillance.
- Disorder problems.
- Detection of abrupt changes.
- Jump detection.
- Fault detection (... FDC).
- ...

My favorite text reference

WOODALL/MONTGOMERY (1999),
Research issues and ideas in statistical process control.
Journal of Quality Technology, 31, 376-386

... in the area of control charting and SPC. As a general definition, we include in this area any statistical method designed to detect changes in a process over time.

Elements of SPC

1 Scope

Statistical process control (SPC) concerns the use of statistical techniques and/or stochastic control algorithms to achieve one or more of the following objectives:

a) to increase knowledge about a process;
b) to steer a process to behave in the desired way;
c) to reduce variation of final-product parameters, or in other ways improve performance of a process.

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3. misses the point: actually, SPC is deployed for checking stability – stability is currently a big topic in semiconductor industry,
4. does not fit to the usual understanding of the term (SPC = control charting).

Further Literature

- WOODALL (2000), Controversies and Contradictions in Statistical Process Control, *Journal of Quality Technology, 32, 341-350*.
- STOUMBOS/REYNOLDS JR./RYAN/WOODALL (2000), The state of statistical process control as we proceed into the 21st century, *J. Amer. Statist. Assoc., 95, 992-997*.

Opinions of my non-statisticians collegues

- "SPC, that is the application of these sophisticated trend rules like 2 of 3",
- "my SPC is out of SPEC",
- "oh, this alarm is caused by a mask that should not be considered in that chart at all",
- "why should I use these $3\,\sigma$ limits if I have no idea what to do after an alarm",
- "control charts should be generally handled by the cleanroom (= shop floor) personnel",
- ...

Focus on Runs Rules Phenomenon

Control Charts History

1. $p, \bar{X}$ control chart designed by Shewhart (1924/31).
2. Bayesian approach by Girshick/Rubin (1952).
3. CUSUM by Page (1954).
4. Runs Rules
   (Page 1955, Western Electric 1956, Roberts 1958, Nelson 1984).
5. EWMA (GMA) by Roberts (1959).

Which charts are available in SPC software?

- WinSPC (AMTC): all flavours of Shewhart charts + large assortment of runs rules,
- SPACE (IFX,Qimonda[,Atmel,TOPPAN,ZMD,Samsung,ST]): Shewhart charts, runs rules, MA, EWMA,
- `ASPECT (AMD): like WinSPC,`
- ...

Runs Rules

Given that $\{X_i\}$ follows $\mathcal{N}(\mu, \sigma^2)$:

Flag if

- (Shewhart limits) $|X_i - \mu_0| > 3\,\sigma$,
- (2 of 3) 2 of 3 succeeding X_i are $\begin{cases} > \mu_0 + 2\,\sigma \\ < \mu_0 - 2\,\sigma \end{cases}$,
- (4 of 5) 4 of 5 succeeding X_i are $\begin{cases} > \mu_0 + \sigma \\ < \mu_0 - \sigma \end{cases}$,
- (8 of 8) 8 of 8 succeeding X_i are $\begin{cases} > \mu_0 \\ < \mu_0 \end{cases}$.

Fall and rise of Runs Rules control charts

1. **Fall:** Champ/Woodall (1987), Exact results for Shewhart control charts with supplementary runs rules, *Technometrics, 29, 393-399*

   It has been shown that supplementary runs rules cause the Shewhart chart to be more sensitive to small shifts in the mean, but not as sensitive as the CUSUM chart.

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2. **Rise**: quality manuals & circles, SPC procedures etc. in practice, Current Index of Statistics: 8 papers in 2002 and newer (Quality Engineering, CSSC)

   KLEIN (2000), Two alternatives to the Shewhart $\bar{X}$ control chart, *JQT, 32, 427-431.*

   YASUI/OJIMA/SUZUKI (2006), Generalization of the run rules for the Shewhart control charts, *Frontiers in Statistical Quality Control, 8, 207-219 – Lenz/Wilrich (Eds.)*

Klein (2000), JQT

Alternative control chart methodologies have been suggested, for example, the CUSUM and EWMA schemes.

Both of these have excellent small process average shift detection capabilities, as described in Montgomery (1997).

However, so far, they do not seem to have achieved widespread application beyond the chemical process industries.

This may be due to a perception that the required calculations are too complex for typical shop floor work and/or the usual organizational inertia associated with procedural changes.

Do the comparison again

- Measure the performance of Shewhart control charts with runs rules, of CUSUM and of EWMA in terms of the
  - Average Run Length (ARL) – average number of control chart points until signal starting from a standard value,
  - steady-state ARL – the same while starting after a longer in-control period.

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    - Average Run Length (ARL) – average number of control chart points until signal starting from a standard value,
    - steady-state ARL – the same while starting after a longer in-control period.
- The charts will be calibrated to exhibit similar in-control behavior!!

The competitors

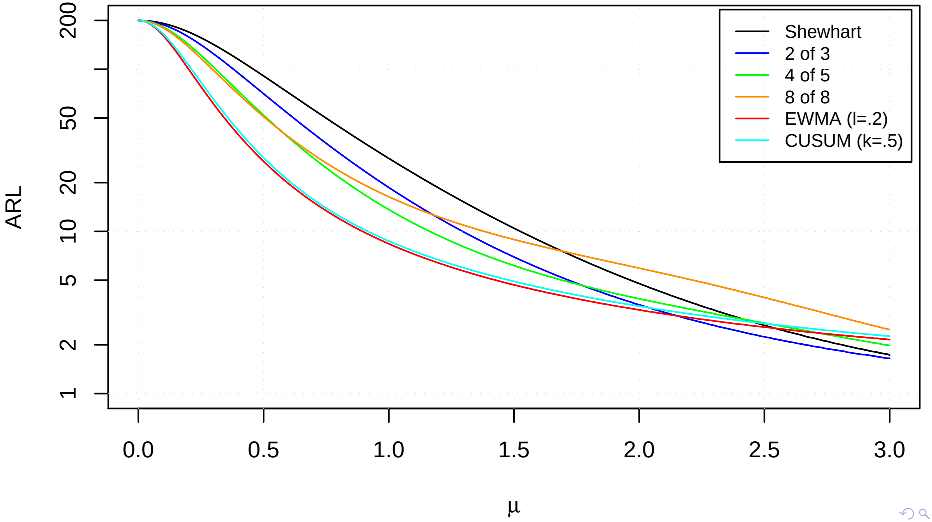
1. CUSUM

$$\begin{aligned} Z_0^{\pm} &= 0\,, \\ Z_i^{\pm} &= \max\{0, Z_{i-1}^{\pm} \pm k\} \qquad \text{with } k = \frac{\mu_0+\mu_1}{2}\,, \\ L &= \min\left\{i \in \mathbb{N} : \max\{Z_i^+, Z_i^-\} > h\,\sigma\right\}\,. \end{aligned}$$

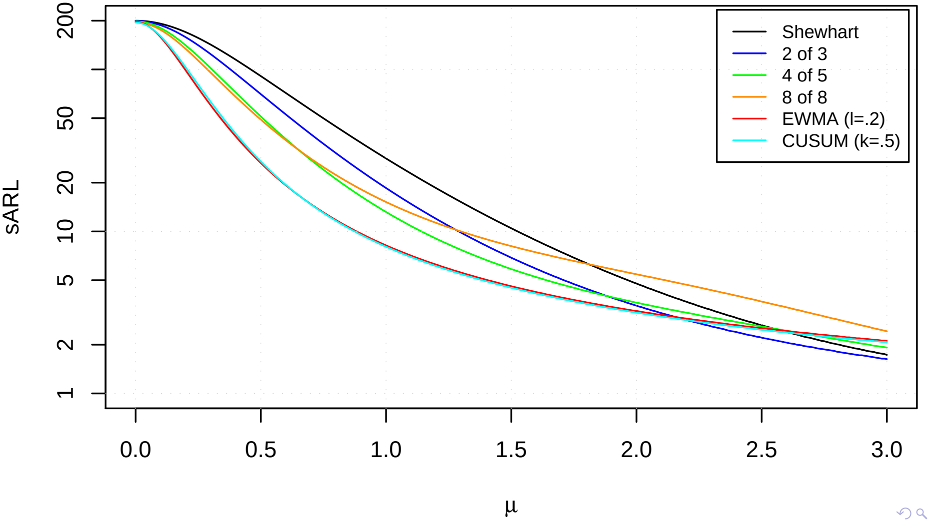
2. EWMA

$$\begin{aligned} Z_0 &= \mu_0\,, \\ Z_i &= (1-\lambda)Z_{i-1} + \lambda X_i \qquad \text{with } \lambda \in (0,1]\,, \\ L &= \min\left\{i \in \mathbb{N} : |Z_i - \mu_0| > c\,\sqrt{\frac{\lambda}{2-\lambda}}\,\sigma\right\}\,. \end{aligned}$$

ARL comparison



Steady-state ARL comparison



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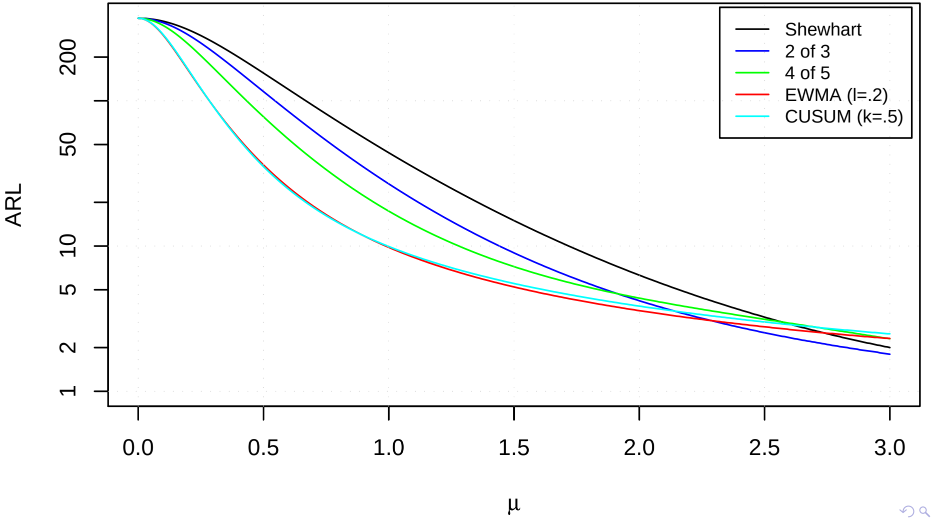

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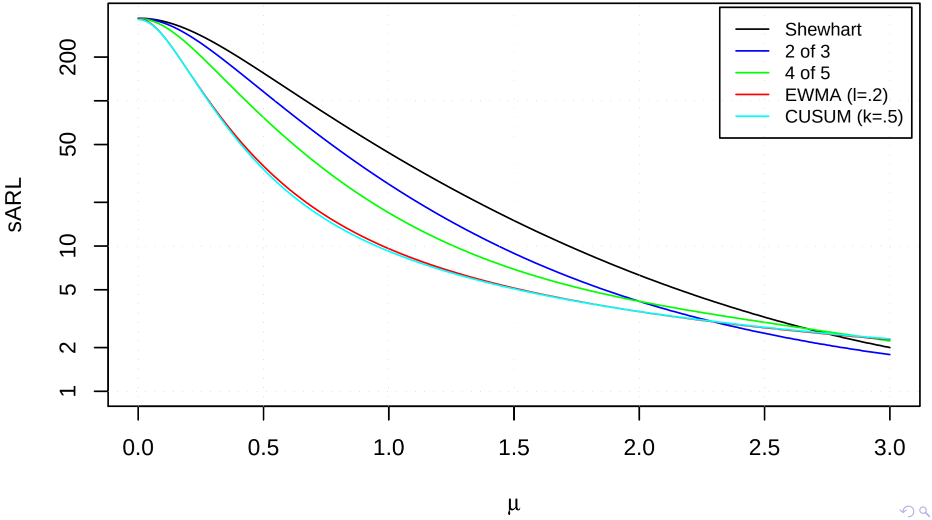


Computational details

- Implementation of Champ/Woodall's algorithm (Markov chain) in R and adaptation of the runs rules zones by a common scaling,
- Simple deployment of the Markov chain for getting the steady-state ARL values,
- CUSUM and EWMA ARL and steady-state ARL values are calculated with the `spc` package in R (numerical solution – Nyström method – of an integral equation).

Why EWMA and not CUSUM (or even Shiryaev-Roberts)?

- EWMA is better known than CUSUM in the semiconductor (and financial) industry,
- EWMA provides an online estimate of the monitored parameter,
- one can facilitate very simple control limits approximation,
- it is easier to sell in practice,
- the performance differences among the three procedures are negligible in practice,
- ...

EWMA supplement – setting control limits

$$UCL = \mu_0 + c \times \sqrt{\frac{\lambda}{2-\lambda}} \times \sigma_0 \qquad \rightarrow \qquad c\ ?$$

- With R sophisticated:

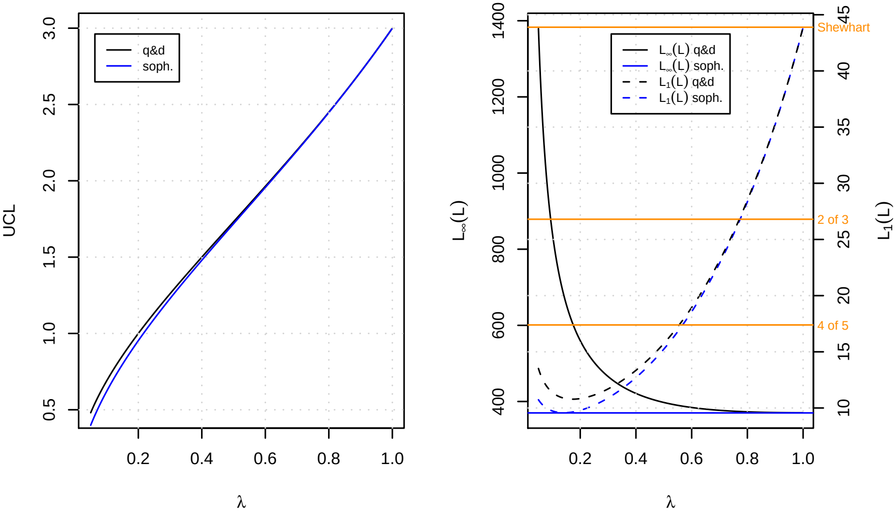
```
library(spc)
xewma.crit(0.2,370,sided="two")
```

  delivers

```
       c
2.858961
```

- Or quick and dirty: $c = 3$

Quick and dirty vs. sophisticated



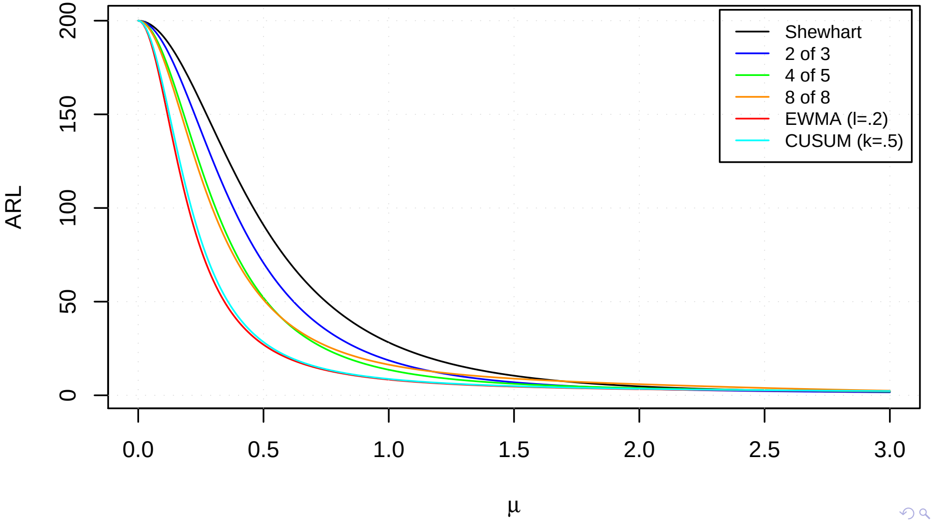
Conclusions

Take the EWMA control chart, because

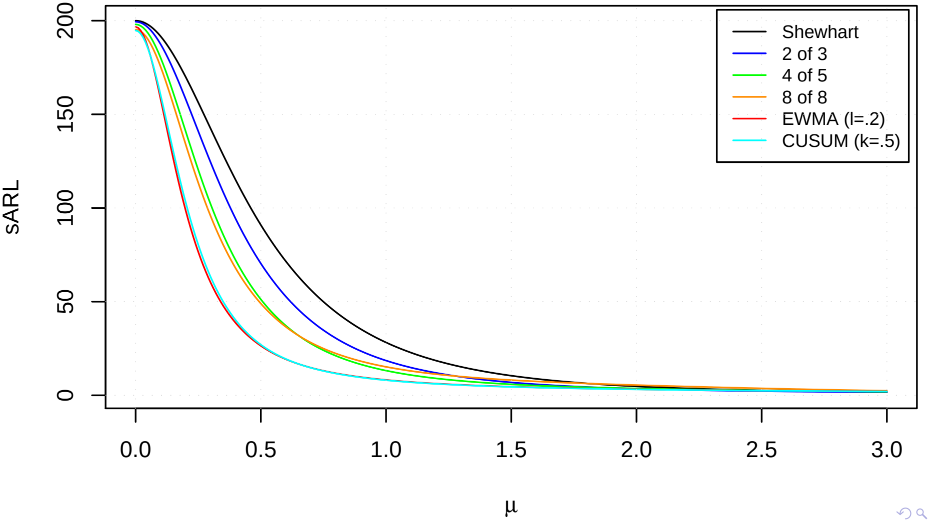
- it behaves as efficient as the optimal ones (CUSUM, SR),
- it beats the Shewhart chart (w/o or w runs rules) for small changes ($\leq 2\,\sigma$),
- it gets a kind of additional Shewhart limit by applying SPEC limits to the original data,
- it allows to clarify the differences between single event analysis and process shift studies,
- it is easy to deploy and to understand,
- the EWMA smoothing is utilized in close application fields like R2R control in semiconductor industry or volatility forecasting in finance,
- ...

Backup

ARL comparison



Steady-state ARL comparison



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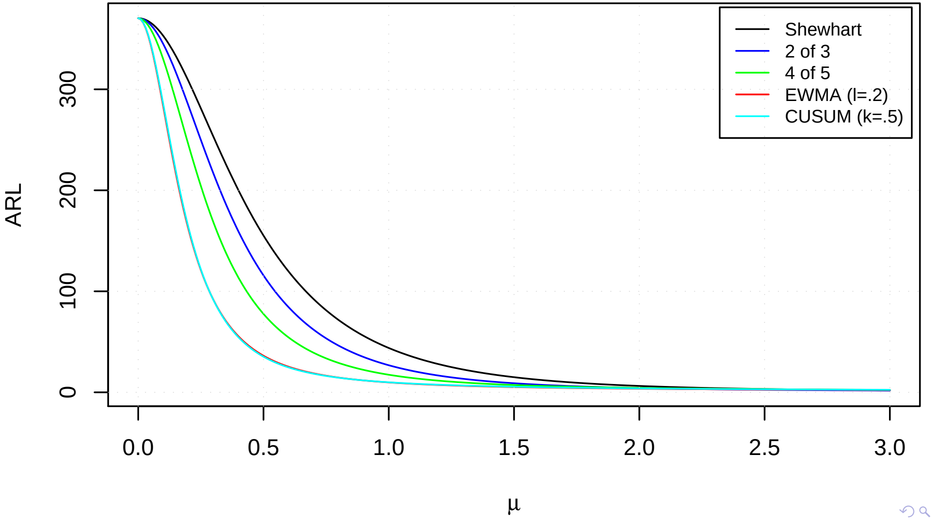

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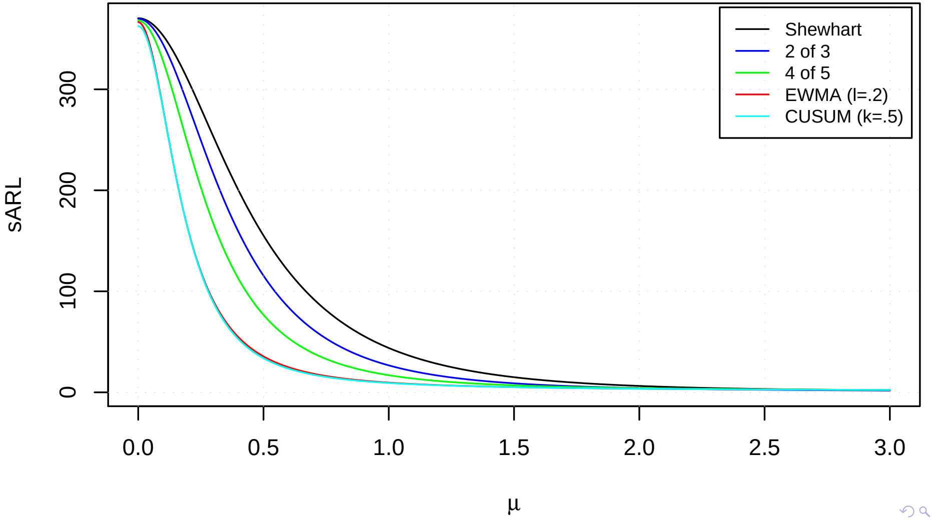

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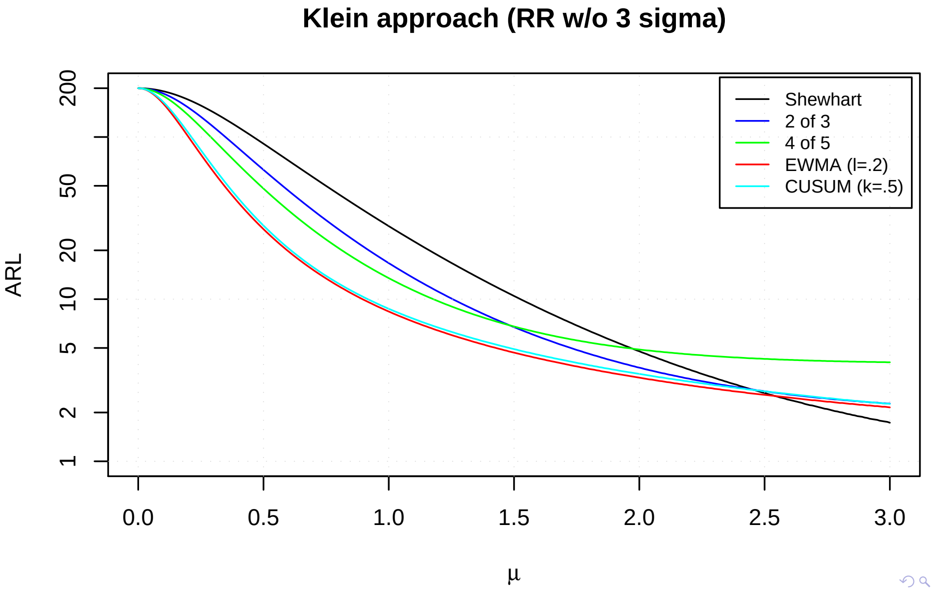

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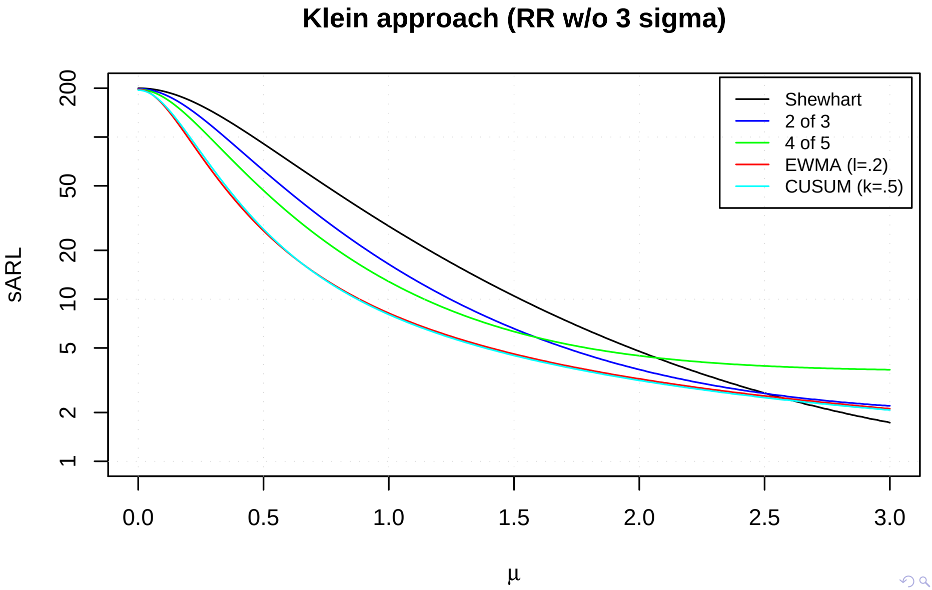

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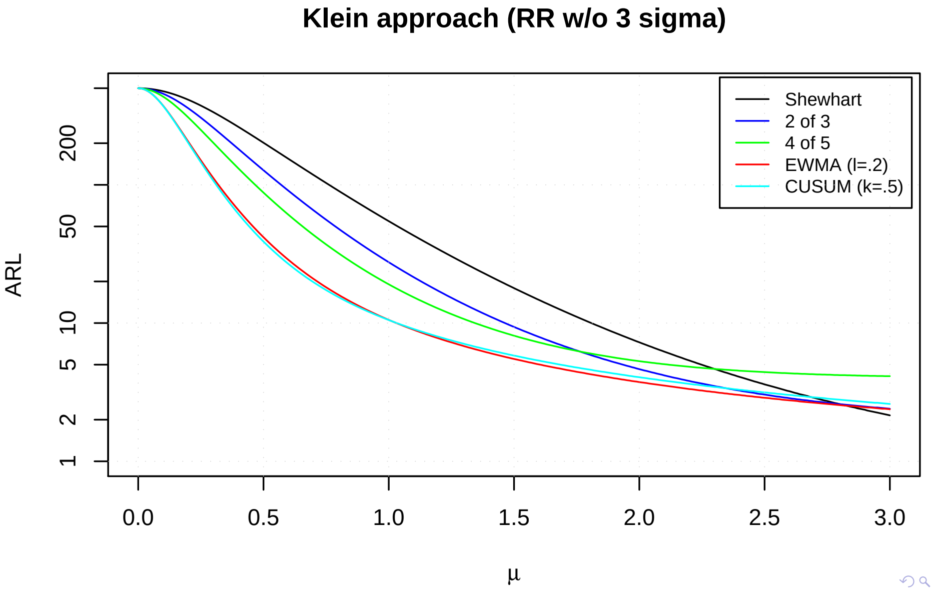

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