R library spc

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Outline

Introduction

1. Control charts are core tools of Statistical Process Control (SPC).
2. Aim: Detect as soon as possible changes in a sequentially observed process with low false alarm rate.
3. Run Length (RL): # of observations until signal.
4. Performance is measured by analyzing the RL.
5. Dominating measure is the Average Run Length (ARL).
6. Charts are designed to meet certain ARL patterns.
7. SPC in practice is usually done with software – feasible software solutions for calculating ARL values and beyond seem to be essential prerequisites.

Control Charts – classics

1. p, $\bar{X}$ control chart designed by Shewhart (1924/31).
2. Bayesian approach by Girshick/Rubin (1952).
3. CUSUM by Page (1954).
4. Runs Rules
   (Page 1955, Western Electric 1956, Roberts 1958, Nelson 1984).
5. EWMA (GMA) by Roberts (1959).

Basic assumptions and notation

- Observations X_i are independent and normally distributed.
- Consider the following change point models:

$$X_i \sim \mathcal{N}(0,1) \quad \text{for} \quad i < \tau\,,$$

and for $i \geq \tau$ (change point τ)

$$X_i \sim \begin{cases} \mathcal{N}(\delta,1) & \text{– shift in mean} \\ \mathcal{N}(0,\beta^2) & \text{– change in scale} \\ \mathcal{N}(\delta,\beta^2) & \text{– mean or scale change} \\ \mathcal{N}((t-\tau+1)\Delta,1) & \text{– drift.} \end{cases}$$

- The above framework is easily adapted to $X_i \sim \mathcal{N}(\mu_0, \sigma_0^2)$, $i < \tau$ etc.

(i) Shewhart chart and (some) Runs Rules

Flag if

- (Shewhart limits) $|X_i| > 3$,
- 2 of 3 succeeding $X_i > 2$ or $X_i < -2$,
- 4 of 5 succeeding $X_i > 1$ or $X_i < -1$,
- 8 of 8 succeeding $X_i > 0$ or $X_i < 0$.

$$L = \inf\{i \in \mathbb{N} : X_i \text{ alone or together with predecessor(s) generates signal.}\}$$

(ii) CUSUM

Originally one-sided

$$Z_0 = z_0 = 0\,,$$
$$Z_i = \max\{0, Z_{i-1} + X_i - k\}$$
$$\text{with} \quad k = \frac{\mu_0 + \mu_1}{2} = \delta/2\,,$$
$$L = \inf\{i \in \mathbb{N} : Z_i > h\}$$

or symmetrically two-sided

$$Z_i^+ = \max\{0, Z_{i-1}^+ + X_i - k\}\,, \quad Z_i^- = \min\{0, Z_{i-1}^- + X_i + k\}\,,$$
$$L = \inf\left\{i \in \mathbb{N} : \max\{Z_i^+, -Z_i^-\} > h\right\}\,.$$

(iii) EWMA

$$
\begin{aligned}
Z_0 &= z_0 = \mu_0 = 0\,, \\
Z_i &= (1-\lambda) Z_{i-1} + \lambda X_i \\
&\quad \text{with } \lambda \in (0,1]\,,
\end{aligned}
$$

$$
L = \min\left\{ i \in \mathbb{N} : |Z_i| > c\sqrt{\frac{\lambda}{2-\lambda}} \right\}
$$

$$
\text{or} \quad L = \min\left\{ i \in \mathbb{N} : |Z_i| > c\sqrt{\frac{\lambda}{2-\lambda}\Big(1-(1-\lambda)^{2i}\Big)} \right\}.
$$

- $E_\tau(\ldots)$ – expectation of ... for given change point τ,
- zero-state ARL:

$$\mathcal{L} = \begin{cases} E_1(L) & ,\tau = 1 \text{ (``early'' change)} \\ E_\infty(L) & ,\tau = \infty \text{ (no change)} \end{cases} .$$

- sequence of conditional delays (medium late changes):

$$D_\tau = E_\tau\big(L - \tau + 1 \mid L \geq \tau\big)\,, \quad \tau = 1, 2, \ldots\,, \quad D_1 = E_1(L)\,.$$

- steady-state ARL (very late change):

$$\mathcal{D} = \lim_{\tau \to \infty} D_\tau\,.$$

- full versions (for mean): $\mathcal{L} = \mathcal{L}(\mu, z_0)$, $D_\tau = D_\tau(\mu_0, \mu_1)$.

ARL $\mathcal{L}$

some glosses

- Introduced by Page (1954) – CUSUM.

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  If it were thought worthwile one could use methods analogous to these given by Page (1954) and estimate the average run length as a function of the departure from the target value. However, as I have already indicated, such computations could be regarded as having the function merely of avoiding unemployment amongst mathematicians.

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- Early calculations:
  - Shewhart charts: $L \sim$ geometric distribution – Page (1954),
  - CUSUM: $\mathcal{L}$ integral equation established in Page (1954); Page (1963) solved a similar one (for range CUSUM) with product mid point rule; Kemp (1958), Ewan/Kemp (1960) provided first reasonable approximations.
  - EWMA: Roberts (1959) deployed Monte Carlo (25,000 replicates).

SPC or statistics software

that is able to calculate $\mathcal{L}$

- SAS: $\mathcal{L}$ for two-sided EWMA and one- and two-sided CUSUM.
- STATISTICA, MINITAB: $\mathcal{L}$ for MEWMA (and MCUSUM).
- statgraphics centurion: similar to SAS.

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- `ANYARL.EXE` from Hawkins (1992): CUSUM, $\mathcal{L}$, $\mathcal{L}(z_0)$, $\mathcal{D}$ for normal, gamma, Poisson, and binomial data; Markov chain + Richardson extrapolation.
- `SEMSTAT.EXE` (SEMATECH): CUSUM + EWMA, $\mathcal{L}$ with integral equation, Markov chain, and diffusion approximation (Siegmund 1985; only for CUSUM) method.
- `DATAPLOT` (NIST): CUSUM, $\mathcal{L}$, collocation (Fellner 1990).

STATLIB/JQT software

that is able to calculate $\mathcal{L}$ or similar

#	author(s)	title	reference
25-3	Gan	... RL Distribution ... CUSUM	1993, 25 (3), 205-215
26-2	Gan, Choi	... ARLs for Exponential CUSUM ...	1994, 26 (2), 134-143
28-3	White, Keats	ARLs and ... Poisson CUSUM	1996, 28 (3), 363-369
31-1	Bodden, Rigdon	... ARL ... MEWMA ...	1999, 31 (1), 120-123
31-2	Davis	... S-Charts ... ARL ...	1999, 31 (2), 246-248
32-2	Gan, Chang	... ARLs of Exponential EWMA ...	2000, 32 (2), 183-187
33-4	Molnau et al.	... ARL ... MEWMA ...	2001, 33 (4), 515-521
34-2-1	Luceno, Puig-Pey	... RL Distribution for CUSUM ...	2002, 34 (2), 209-215

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And R

is able to calculate $\mathcal{L}$

- R
  - dialect of S (Bell Labs, 1976, 1988, 1998; Insightful & S-PLUS, ...),
  - GNU project (1991 start, 1993 public, 1995 GPL),
  - `http://www.r-project.org/`,
  - Win, Mac, Unix,
  - current version 2.11.1, (2000 1.0.0)

(I) Two famous papers coded in R.

(II) R library spc (2004 0.1, 2009 0.3, under preparation 0.4)

(I) Two famous papers coded in R

- Brook/Evans (1972), *An approach to the probability distribution of CUSUM run length*, Biometrika 59, 539-549.

- Lucas/Saccucci (1990), *Exponentially weighted moving average control schemes: Properties and enhancements*, Technometrics 32, 1-12.

Brook/Evans (1972)

- *Idea*: Replace the continuous chart statistic Z_i with a Markov chain (MC) and utilize results from MC theory.
- It is one of the most frequently cited SPC papers.
- Parameters are k, h, z_0, μ and matrix dimension t.

$$Z_0 = z_0\,,\; Z_i = \max\{0, Z_{i-1} + X_i - k\}\,,$$
$$L = \inf\{i \in \mathbb{N} : Z_i > h\}\;,\; \mathcal{L} = E_\mu(L)\,.$$

Brook/Evans (1972)

with R

```
BE.cusum.arl <- function(k, h, mu, z0=0, t=15) {
  i <- 1:(t-1)
  w <- 2*h/(2*t-1)
  qij <- function(i,j)
         pnorm((j-i)*w+w/2, mean=mu-k) - pnorm((j-i)*w-w/2, mean=mu-k)
  Qi <- function(i) pnorm( -i*w+w/2, mean=mu-k)
  Q <- rbind( cbind( Qi(0), t(qij(0,i)) ),
              cbind( Qi(i), outer(i,i,qij) ) )
  one <- array(1,t)
  I <- diag(1,t)
  ARL <- solve(I-Q,one)
  arl <- 1 + Qi(z0*w)*ARL[1] + sum(qij(z0*w,i)*ARL[-1])
  arl
}
```

Brook/Evans (1972) with R

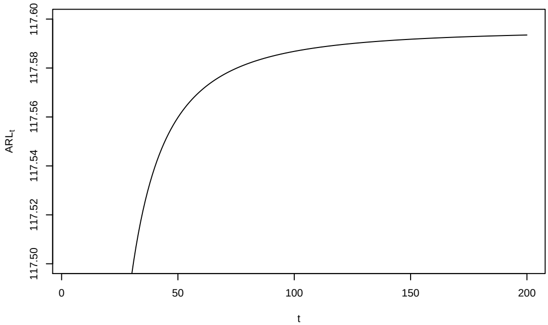
```
k <- 0.5
h <- 3

BE.cusum.arl(k, h, 1.5, t=5)
BE.cusum.arl(k, h, 0, t=5)
BE.cusum.arl(k, h, 0, t=15)
```

```
3.77
113.47
117.18
```

Brook/Evans (1972) with R

```
T.c.be.arl <- Vectorize("BE.cusum.arl", "t")
ts <- 4:200
be.arls <- T.c.be.arl(k, h, 0, t=ts)
plot(ns, be.arls, type="l", xlab="t",
     ylab=expression(ARL[t]), ylim=c(117.5, 117.6))
```



Lucas/Saccucci (1990)

- *Idea*: same as in Brook/Evans (1972).
- It is (also) one of the most frequently cited SPC papers.
- Parameters are λ, c, z_0, μ and half matrix dimension t.

$$Z_0 = z_0\,,\; Z_i = (1-\lambda) Z_{i-1} + \lambda X_i$$

$$L = \min\left\{ i \in \mathbb{N} : |Z_i| > c\sqrt{\frac{\lambda}{2-\lambda}} \right\}\,,\; \mathcal{L} = E_\mu(L)\,.$$

- Algorithmic/computational details are published in:
  Lucas/Saccucci (1990), ARLs for EWMA Control Schemes Using the Markov Chain Approach, JQT 22, 154-162.

Lucas/Saccucci (1990) with R

```
LS.ewma.ARL <- function(l, c, mu, z0=0, t=50) {
  c <- c*sqrt(l/(2-l))
  i <- (-t:t)
  w <- 2*c/(2*t+1)
  qij <- function(i,j)  pnorm((j*w-(1-l)*i*w+w/2)/l, mean=mu) -
                        pnorm((j*w-(1-l)*i*w-w/2)/l, mean=mu)
  Q <- outer(i,i,qij)
  one <- array(1,2*t+1)
  I <- diag(1,2*t+1)
  ARL <- solve(I-Q,one)
  arl <- 1 + sum(qij(z0*w,i)*ARL)
  arl
}
```

Lucas/Saccucci (1990) with R

```
fast.LS.ewma.arl <- function(l, c, mu, z0=0, t=35) {
  T.LS.ewma.arl <- Vectorize("LS.ewma.arl", "t")
  ts <- t-8*(0:4)
  L <- T.LS.ewma.arl(l, c, mu, z0=z0, t=ts)
  LM <- lm(L ~ I(1/ts) + I(1/ts^2))
  rARL <- coef(LM)[1]
  names(rARL) <- NULL
  rARL
}
```

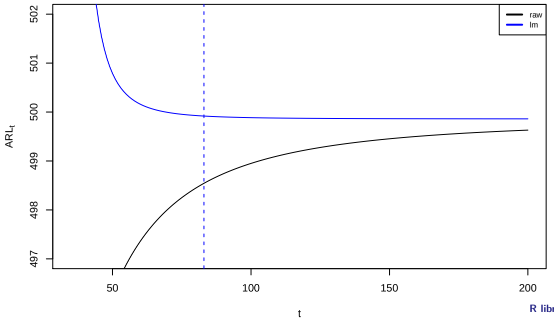
Lucas/Saccucci (1990) with R

```
fM.e.ls.arl <- Vectorize("fast.LS.ewma.arl", "mu")
mus <- c( (0:4)/4, (3:8)/2, 5)
lambda <- 0.03
c <- 2.437
arls <- round(fM.e.ls.arl(lambda, c, mu=mus, t=83), digits=2)
cbind(mus, arls)
```

```
       mus   arls
 [1,] 0.00 499.92
 [2,] 0.25  76.73
 [3,] 0.50  29.32
 [4,] 0.75  17.63
 [5,] 1.00  12.60
 [6,] 1.50   8.07
 [7,] 2.00   5.99
 [8,] 2.50   4.80
 [9,] 3.00   4.03
[10,] 3.50   3.49
[11,] 4.00   3.11
[12,] 5.00   2.55
```

Lucas/Saccucci (1990) with R

```
T.e.ls.arl  <- Vectorize("LS.ewma.arl", "t")
fT.e.ls.arl <- Vectorize("fast.LS.ewma.arl", "t")
ts <- 35:200
ls.arls  <- T.e.ls.arl(lambda, c, 0, t=ts)
f.ls.arls <- fT.e.ls.arl(lambda, c, 0, t=ts)
plot(ts, ls.arls, type="l",
     xlab="t", ylab=expression(ARL[t]), ylim=c(497, 502))
lines(ts, f.ls.arls, col="blue")
```



- ARL profiles for two-sided charts,
- ARL profiles for one-sided charts,
- setup and display of several two-sided charts.

Numerical Methods

in a nutshell

- Eventually, there is a linear equation system to solve.
- Some sophisticated stochastic approximations are at work such as:
  - Edgeworth expansion (Robinson/Ho 1978),
  - diffusion approximation (Reynolds 1975, Siegmund 1985, Srivastava/Wu 1993, ...),
  - van Dobben de Bruyn's (1968) list: Neumann series, Kemp (1961), Wiener-Hopf techniques, renewal theory,
  - improved Wald approximations (Khan 1979),
  - special ones such as Lorden/Eisenberger (1973), Tartakovsky/Ivanova (1992), ...
  - ...
- Analytic solutions (exponential CUSUM – Vardeman/Ray 1985, exponential EWMA – Gan/Chang 2000, Erlang CUSUM – Knoth 1998, ...).

Among all algorithms where

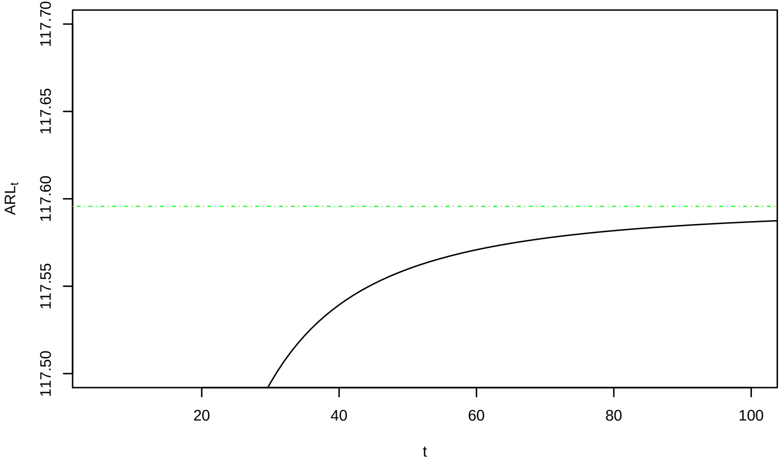
eventually is a linear equation system to solve
we want to identify "the" algorithm.

⇝ Do a "small" study.

- Dimension of the resulting matrix is a rough speed index.
- Starting point is Brook/Evans (1972) for CUSUM ARL.
- Finally, the choice for the R library spc is justified.

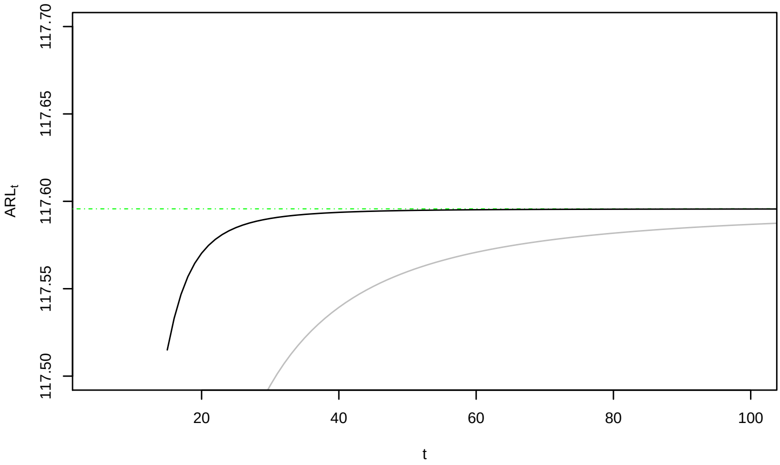
CUSUM $\mathcal{L}$ – numerical Methods

Brook/Evans (1972), Page (1963) – product midpoint rule



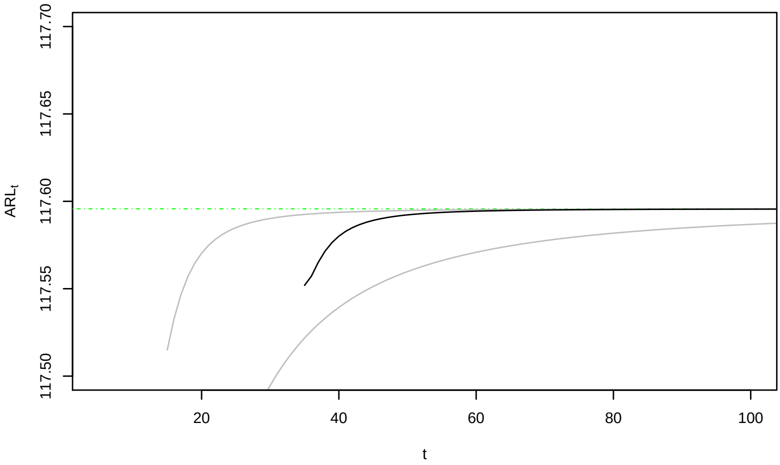
CUSUM $\mathcal{L}$ – numerical Methods

Brook/Evans (1972) – accelerate with simple extrapolation



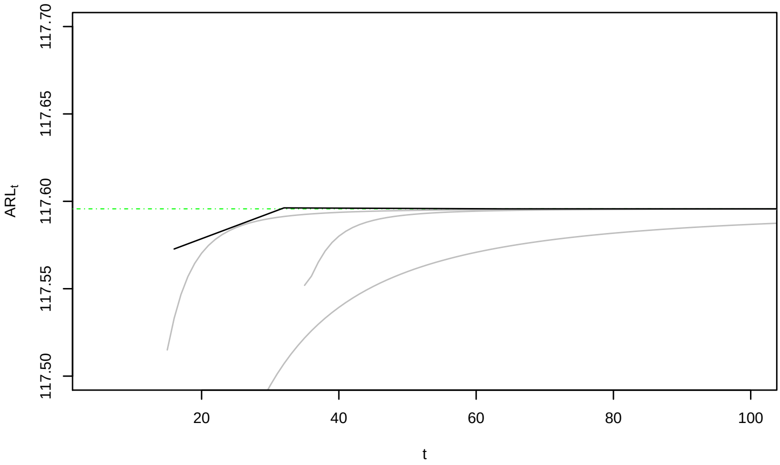
CUSUM $\mathcal{L}$ – numerical Methods

Lucas/Saccucci (1990) – accelerate with linear regression



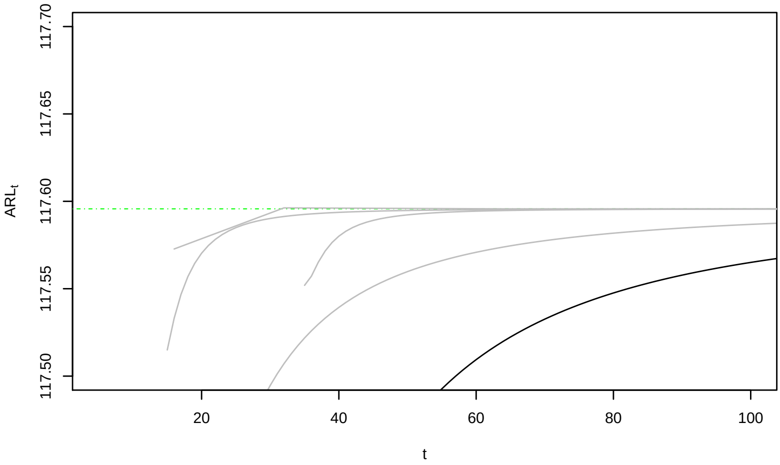
CUSUM $\mathcal{L}$ – numerical Methods

Hawkins (1992) – accelerate with Richardson extrapolation



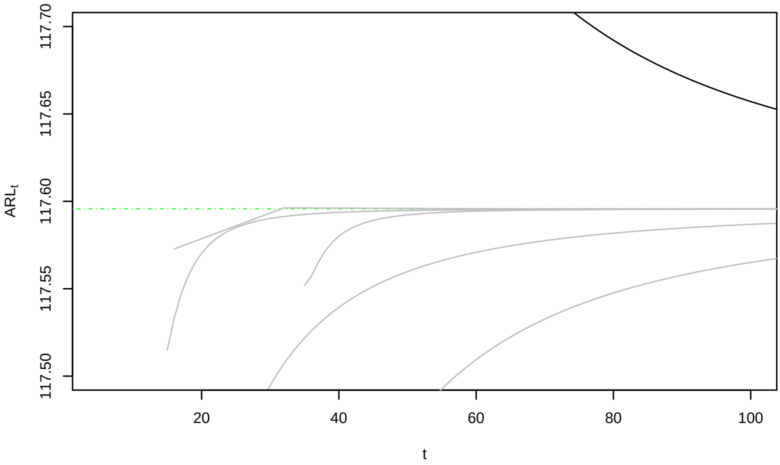
CUSUM $\mathcal{L}$ – numerical Methods

Nyström (integral equation + quadrature) – mid point rule



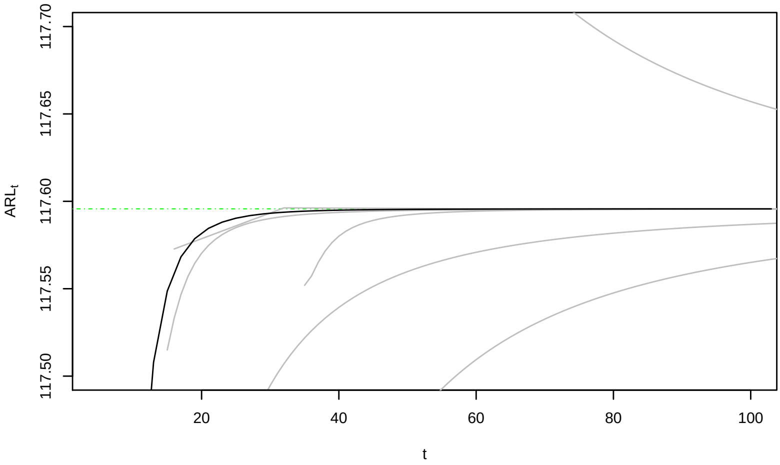
CUSUM $\mathcal{L}$ – numerical Methods

Nyström – trapezoid rule



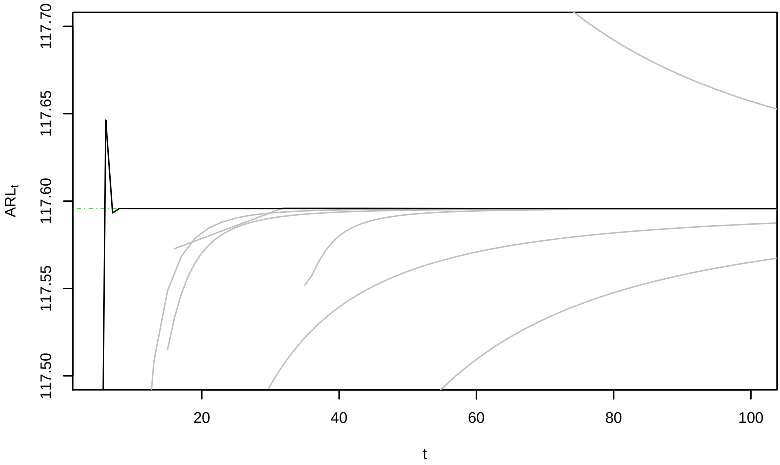
CUSUM $\mathcal{L}$ – numerical Methods

Nyström – Simpson rule



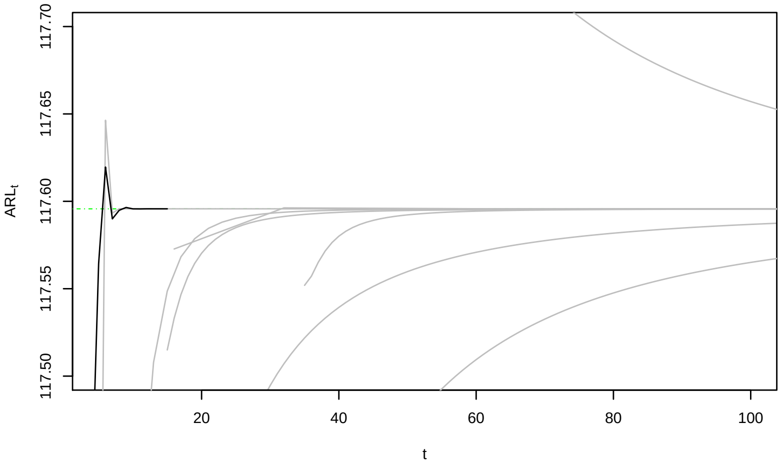
CUSUM $\mathcal{L}$ – numerical Methods

Nyström – Gauss-Legendre rule (Lucas 1976, Crowder 1987)



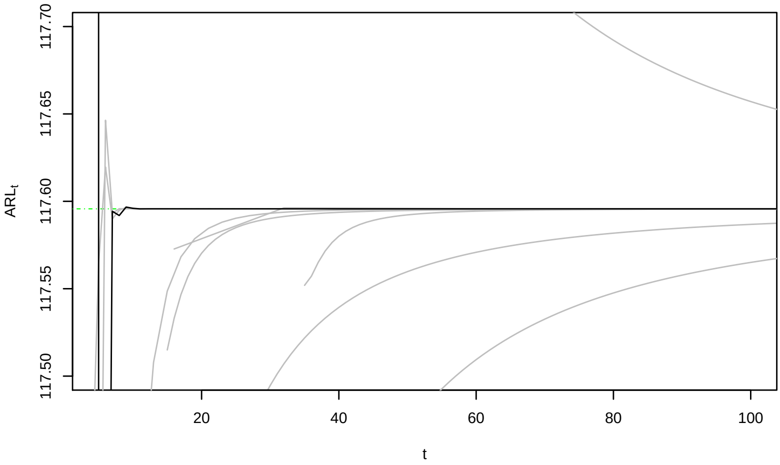
CUSUM $\mathcal{L}$ – numerical Methods

integral equation + collocation (Fellner 1990)



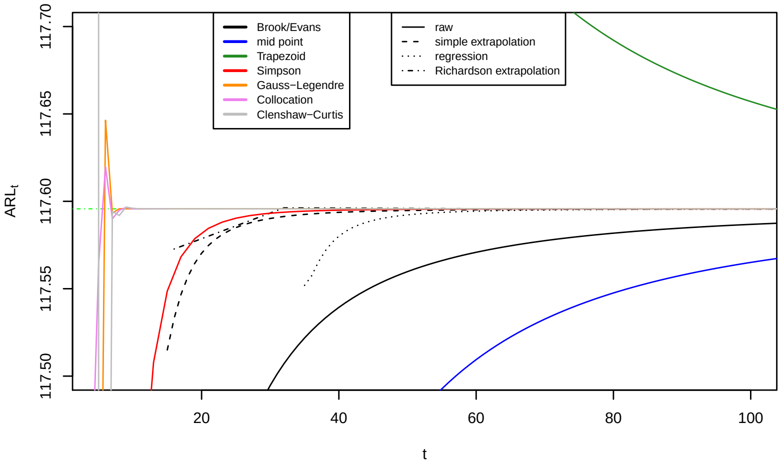
CUSUM $\mathcal{L}$ – numerical Methods

Nyström – Clenshaw-Curtis rule (Capizzi/Masarotto 2010)



CUSUM $\mathcal{L}$ – numerical Methods

all together



CUSUM $\mathcal{L}$ – numerical Methods

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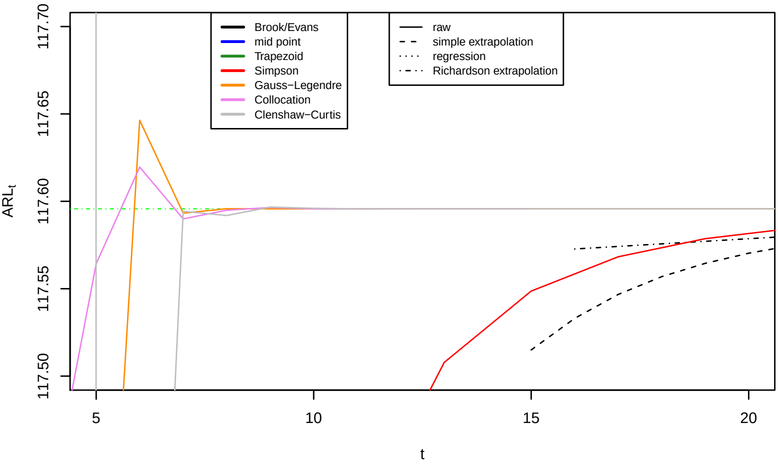


Decision

Take Gauss-Legendre whenever it is possible/appropriate!

Library spc

Some details etc.

Library spc

Overview

- Allows to calculate the
- zero-state and steady-state ARL
- of Shewhart, Shewhart with some Runs Rules, CUSUM, EWMA (and Shiryaev-Roberts)
- for step change and drift of the mean and scale change.

Library spc

List of commands

name	characteristic	method	note
`xcusum.arl`	$\mathcal{L}$	GL	1-sided, 2-sided, Crosier
`xcusum.ad`	$\mathcal{D}$	...	...
`xewma.arl`	$\mathcal{L}$, D_τ	GL	1-sided, 2-sided, fixed, varying limits
`xewma.ad`	$\mathcal{D}$	...	...
`xgrsr.arl`	$\mathcal{L}$	GL	1-sided
`xgrsr.ad`	$\mathcal{D}$	...	...
`xshewhartrunsrules.arl`	$\mathcal{L}$	MC	2-sided
`xshewhartrunsrules.ad`	$\mathcal{D}$	...	...
`sewma.arl`	$\mathcal{L}$	coll	1-sided, 2-sided, Knoth (2005)
`xsewma.arl`	$\mathcal{L}$	coll/GL	1-sided, 2-sided, Knoth (2007)
`xDcusum.arl`	$\mathcal{L}$, D_τ	GL	1-sided, drift
`xDewma.arl`	$\mathcal{L}$, D_τ	GL	1-sided, 2-sided, fixed limits, drift
`xDgrsr.arl`	$\mathcal{L}$, D_τ	GL	1-sided, drift
`xDshewhartrunsrules.arl`	$\mathcal{L}$	MC	2-sided, drift

link	R function	what	who	note
1	`xcusum.arl`	$\mathcal{L}$	Brook/Evans (1972)	
2	`xcusum.arl`	$\mathcal{L}$	Lucas/Crosier (1982)	
3	`xcusum.ad`	$\mathcal{D}$	Crosier (1986)	
4	`xewma.arl`	$\mathcal{L}$	Crowder (1987)	
5	`xewma.arl`	$\mathcal{L}$	Lucas/Saccucci (1990)	fir
6	`xewma.arl`	$\mathcal{L}$	Rhoads/Montgomery/Mastrangelo (1996)	vacl
7	`xewma.arl`	D_τ	Knoth (2003)	vacl
8	`xgrsr.arl`	$\mathcal{L}, \mathcal{D}$	Moustakides/Polunchenko/Tartakovsky (2009)	SR
9	`xs..rules.arl`	$\mathcal{L}$	Champ/Woodall (1987)	
A	`sewma.arl`	$\mathcal{L}$	Knoth (2005)	upper
B	`sewma.arl`	$\mathcal{L}$	Knoth (2005)	Rupper
C	`sewma.arl`	$\mathcal{L}$	Knoth (2005)	lower
D	`xsewma.arl`	$\mathcal{L}$	Knoth (2007)	
E	`xDewma.arl`	$\mathcal{L}$	Gan (1991)	drift
E	`xDs..rules.arl`	$\mathcal{L}$	Aerne/Champ/Rigdon (1991)	drift

Brook/Evans (1972) with R

▸ All

```
k <- 0.5
h <- 3
xcusum.arl(k, h, 0)
```

```
     arl
117.5957
```

Lucas, Crosier (1982) with R

All

```
k <- .5
h <- 4
mu <- c(0, .25, .5, .75, 1, 1.5, 2, 2.5, 3, 4, 5)
MU.xcusum.arl <- Vectorize("xcusum.arl", "mu")
arl1 <- MU.xcusum.arl(k, h, mu=mu, sided="two")
arl2 <- MU.xcusum.arl(k, h, mu=mu, hs=h/2, sided="two")
round(cbind(mu, arl1, arl2), digits=2)
```

```
      mu   arl1   arl2
arl 0.00 167.68 148.70
arl 0.25  74.22  62.70
arl 0.50  26.63  20.06
arl 0.75  13.29   8.97
arl 1.00   8.38   5.29
arl 1.50   4.75   2.86
arl 2.00   3.34   2.01
arl 2.50   2.62   1.59
arl 3.00   2.19   1.33
arl 4.00   1.71   1.07
arl 5.00   1.31   1.01
```

▸ All

```
k <- .5
h2 <- 4
hC <- 3.73
mu <- c(0, .25, .5, .75, 1, 1.5, 2, 2.5, 3, 4, 5)
MU.xcusum.ad <- Vectorize("xcusum.ad", "mu1")
ad2 <- MU.xcusum.ad(k, h2, mu1=mu, sided="two", r=20)
adC <- MU.xcusum.ad(k, hC, mu1=mu, sided="Crosier")
round(cbind(mu, ad2, adC), digits=2)
```

```
     mu    ad2    adC
ad 0.00 162.29 164.65
ad 0.25  71.51  69.07
ad 0.50  25.24  24.37
ad 0.75  12.37  12.17
ad 1.00   7.72   7.70
ad 1.50   4.33   4.40
ad 2.00   3.05   3.12
ad 2.50   2.39   2.47
ad 3.00   2.01   2.07
ad 4.00   1.55   1.60
ad 5.00   1.22   1.29
```

▸ All

```
l1 <- .5
l2 <- .05
c <- 2
mu <- (0:16)/4
MU.xewma.arl <- Vectorize("xewma.arl", "mu")
arl1 <- MU.xewma.arl(l1, c, mu=mu, sided="two")
arl2 <- MU.xewma.arl(l2, c, mu=mu, sided="two")
round(cbind(mu, arl1, arl2), digits=2)
```

```
      mu  arl1   arl2
arl 0.00 26.45 127.53
arl 0.25 20.12  43.94
arl 0.50 11.89  18.97
arl 0.75  7.29  11.64
arl 1.00  4.91   8.38
arl 1.25  3.59   6.56
arl 1.50  2.80   5.41
arl 1.75  2.29   4.62
arl 2.00  1.95   4.04
arl 2.25  1.70   3.61
arl 2.50  1.51   3.26
```

▸ All

```
l1 <- .5
l2 <- .03
c1 <- 3.071
c2 <- 2.437
hs1 <- c1/2
hs2 <- c2/2
mu <- c(0, .5, 1, 2, 3, 5)
MU.xewma.arl <- Vectorize("xewma.arl", "mu")
arl1 <- MU.xewma.arl(l1, c1, mu=mu, hs=hs1, sided="two", limits="fir")
arl2 <- MU.xewma.arl(l2, c2, mu=mu, hs=hs2, sided="two", limits="fir")
round(cbind(mu, arl1, arl2), digits=2)
```

```
     mu   arl1   arl2
arl 0.0 493.03 404.59
arl 0.5  85.89  18.40
arl 1.0  15.91   7.34
arl 2.0   2.87   3.43
arl 3.0   1.45   2.34
arl 5.0   1.01   1.57
```

Rhoads, Montgomery, Mastrangelo (1996) with R

▸ All

```
l <- .03
c <- 2.437
mu <- c(0, .5, 1, 1.5, 2, 3, 4)
sl <- sqrt(l*(2-l))
arlfix  <- sapply(mu,l=l,c=c,sided="two",xewma.arl)
arlvacl <- sapply(mu,l=l,c=c,sided="two",limits="vacl",xewma.arl)
arlfir  <- sapply(mu,l=l,c=c,hs=c/2,sided="two",limits="fir",xewma.arl)
arlboth <- sapply(mu,l=l,c=c,hs=c/2*sl,sided="two",limits="both",xewma.arl)
round(cbind(mu, arlfix, arlvacl, arlfir, arlboth), digits=1)
```

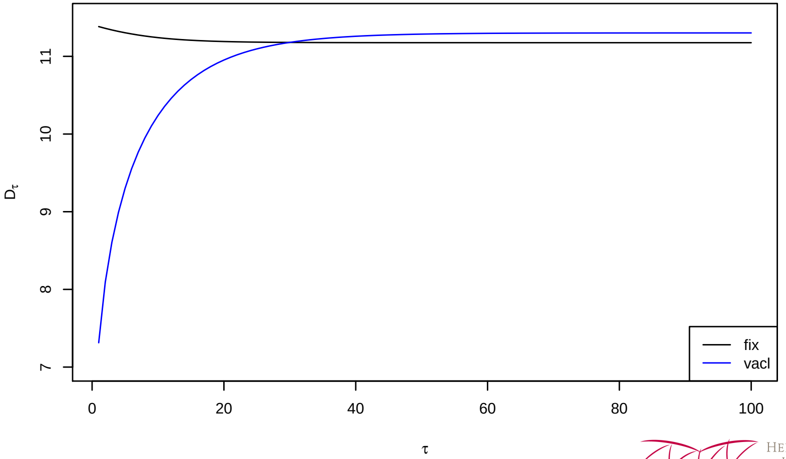
```
     mu arlfix arlvacl arlfir arlboth
arl 0.0  499.9   445.0  404.6   305.4
arl 0.5   29.3    20.4   18.4    12.6
arl 1.0   12.6     6.4    7.3     3.7
arl 1.5    8.1     3.4    4.6     1.9
arl 2.0    6.0     2.2    3.4     1.3
arl 3.0    4.0     1.3    2.3     1.0
arl 4.0    3.1     1.1    1.9     1.0
```

▸ All

```
Q.xewma.arl <- Vectorize("xewma.arl", "q")
lambda <- 0.05
L0 <- 500
c1 <- xewma.crit(lambda, L0, sided="two")
c2 <- xewma.crit(lambda, L0, sided="two", limits="vacl")
qs <- 1:100
D1 <- Q.xewma.arl(lambda, c1, 1, sided="two", q=qs)
D2 <- Q.xewma.arl(lambda, c2, 1, sided="two",
                  limits="vacl", q=qs)
plot(qs, D1, type="l", ylim=c(7, 11.5),
     xlab=expression(tau), ylab=expression(D[tau]))
lines(qs, D2, col="blue")
legend("bottomright", c("fix", "vacl"), lty=1,
       col=c("black", "blue"))
```

D_τ vs. τ with R

▸ All



Moustakides/Polunchenko/Tartakovsky (2009) with R

▸ All

```
Gxgrsr.arl  <- Vectorize("xgrsr.arl", "g")
As <- c(28.02, 56.04, 280.19, 560.37, 2801.75, 5603.7)
gs <- log(As)
theta <- 1
zr <- -6
arls0 <- round(Gxgrsr.arl(theta/2, gs, 0, zr=zr, r=100), digits=2)
arls1 <- round(Gxgrsr.arl(theta/2, gs, theta, zr=zr, r=100), digits=2)
data.frame(As, arls0, arls1)
```

```
       As    arls0 arls1
1   28.02    50.79  5.46
2   56.04   100.79  6.71
3  280.19   500.80  9.78
4  560.37  1000.79 11.14
5 2801.75  5000.61 14.34
6 5603.70 10000.78 15.72
```

▸ All

```
mus <- (0:15)/5
Mxshewhartrunsrules.arl <- Vectorize(xshewhartrunsrules.arl, "mu")
C1 <- round(Mxshewhartrunsrules.arl(mus, type="1"), digits=2)
C12 <- round(Mxshewhartrunsrules.arl(mus, type="12"), digits=2)
C13 <- round(Mxshewhartrunsrules.arl(mus, type="13"), digits=2)
C14 <- round(Mxshewhartrunsrules.arl(mus, type="14"), digits=2)
cbind(mus, C1, C12, C13, C14)
```

```
      mus     C1    C12    C13    C14
 [1,] 0.0 370.40 225.44 166.05 152.73
 [2,] 0.2 308.43 177.56 120.70 110.52
 [3,] 0.4 200.08 104.46  63.88  59.76
 [4,] 0.6 119.67  57.92  33.99  33.64
 [5,] 0.8  71.55  33.12  19.78  21.07
 [6,] 1.0  43.89  20.01  12.66  14.58
 [7,] 1.2  27.82  12.81   8.84  10.90
 [8,] 1.4  18.25   8.69   6.62   8.60
 [9,] 1.6  12.38   6.21   5.24   7.03
[10,] 1.8   8.69   4.66   4.33   5.85
[11,] 2.0   6.30   3.65   3.68   4.89
...
```

Knoth (2005)

with R

▸ All

```
## compare with Table 1 (p. 347): 249.9997
## Monte Carlo with 10^9 replicates: 249.9892 +/- 0.008
l <- .025
df <- 1
cu <- 1 + 1.661865*sqrt(l/(2-l))*sqrt(2/df)
sewma.arl(l, 0, cu, 1, df)
```

```
     arl
249.9997
```

Knoth (2005) with R

▸ All

```
## ARL values for upper and lower EWMA charts with reflecting barriers
## (reflection at in-control level sigma0 = 1)
## examples from Knoth (2005), Tables 4 and 5
Ssewma.arl <- Vectorize("sewma.arl", "sigma")
l <- 0.15
df <- 4
cu <- 1 + 2.4831*sqrt(l/(2-l))*sqrt(2/df)
sigmas <- c(1 + (0:5)/100, 1 + (1:5)/10, 2)
arls <- round(Ssewma.arl(l,1,cu,sigmas,df,sided="Rupper",r=100), digits=2)
cbind(sigmas, arls)
```

```
    sigmas   arls
arl   1.00 100.00
arl   1.01  85.31
arl   1.02  73.35
arl   1.03  63.53
arl   1.04  55.43
arl   1.05  48.68
arl   1.10  27.89
arl   1.20  12.90
arl   1.30   7.86
```

Knoth (2005) with R

▸ All

```
## lower chart with reflection at sigma0=1 in Table 5
## original entries are
l <- 0.115
df <- 5
cl <- 1 - 2.0613*sqrt(l/(2-l))*sqrt(2/df)
sigmas <- c((10:6)/10)
arls <- round(Sseewma.arl(l,cl,1,sigmas,df,sided="Rlower",r=100), digits=2)
cbind(sigmas, arls)
```

```
    sigmas   arls
arl    1.0 200.04
arl    0.9  38.47
arl    0.8  14.63
arl    0.7   8.65
arl    0.6   6.31
```

Knoth (2007) with R

▸ All

```
## collocation results in Table 1
## Monte Carlo with 10^9 replicates: 252.307 +/- 0.0078

mu <- 0
sigma <- 1
df   <- 4   # batch size n=5, df=n-1
lx   <- .134
cx   <- .345476571*sqrt(df+1)/sqrt(lx/(2-lx))
ls   <- .1
csu <- 1 + .477977

xsewma.arl(lx,cx, ls,csu, df, mu,sigma, Nx=25,Ns=25, sided="upper")
```

```
     arl
252.3001
```

Gan (1991) with R

▸ All

```
DxDewma.arl <- Vectorize(xDewma.arl, "delta")
lambda1 <- 0.676
lambda2 <- 0.242
lambda3 <- 0.047
h1 <- 2.204269/sqrt(lambda1/(2-lambda1))
h2 <- 1.110554/sqrt(lambda2/(2-lambda2))
h3 <- 0.402546/sqrt(lambda3/(2-lambda3))
deltas <- c(.0001, .001, .002, .005, .01, .05, .1, 1, 3)
arlE10 <- round(xewma.arl(lambda1, h1, 0, sided="two"), digits=2)
arlE1 <- c(arlE10,
  round(DxDewma.arl(lambda1, h1, deltas, sided="two", with0=TRUE), digits=2))
arlE20 <- round(xewma.arl(lambda2, h2, 0, sided="two"), digits=2)
arlE2 <- c(arlE20,
  round(DxDewma.arl(lambda2, h2, deltas, sided="two", with0=TRUE), digits=2))
arlE30 <- round(xewma.arl(lambda3, h3, 0, sided="two"), digits=2)
arlE3 <- c(arlE30,
  round(DxDewma.arl(lambda3, h3, deltas, sided="two", with0=TRUE), digits=2))
cbind(delta=c(0, deltas), arlE1, arlE2, arlE3)
```

Gan (1991) with R

▸ All

	delta	arlE1	arlE2	arlE3
arl	0e+00	500.00	500.00	500.00
arl	1e-04	481.53	459.58	424.40
arl	1e-03	289.20	230.79	185.50
arl	2e-03	210.18	161.61	129.42
arl	5e-03	126.06	94.60	77.92
arl	1e-02	81.65	61.25	52.72
arl	5e-02	27.45	21.78	21.85
arl	1e-01	17.01	14.20	15.27
arl	1e+00	4.09	4.28	5.25
arl	3e+00	2.60	2.90	3.43

Aerne/Champ/Rigdon (1991) with R

▸ All

```
c1of1 <- 3.069/3
c2of3 <- 2.1494/2
c4of5 <- 1.14
c10   <- 3.2425/3
DxDshewhartrunsrules.arl <- Vectorize(xDshewhartrunsrules.arl, "delta")
deltas <- 10^(-(18:0)/8)
arl1of1 <-
round(DxDshewhartrunsrules.arl(deltas, c=c1of1, type="1"), digits=2)
arl2of3 <-
round(DxDshewhartrunsrules.arl(deltas, c=c2of3, type="12"), digits=2)
arl4of5 <-
round(DxDshewhartrunsrules.arl(deltas, c=c4of5, type="13"), digits=2)
arl10 <-
round(DxDshewhartrunsrules.arl(deltas, c=c10, type="SameSide10"), digits=2)
cbind(delta=round(deltas, digits=6), arl1of1, arl2of3, arl4of5, arl10)
```

Aerne/Champ/Rigdon (1991) with R

▸ All

	delta	arl1of1	arl2of3	arl4of5	arl10
[1,]	0.005623	136.66	120.89	105.32	107.07
[2,]	0.007499	114.97	101.22	88.08	89.93
[3,]	0.010000	96.02	84.21	73.30	75.23
[4,]	0.013335	79.68	69.67	60.74	62.72
[5,]	0.017783	65.75	57.37	50.17	52.18
[6,]	0.023714	53.99	47.06	41.33	43.35
[7,]	0.031623	44.15	38.47	33.98	35.99
[8,]	0.042170	35.96	31.36	27.91	29.90
[9,]	0.056234	29.20	25.50	22.91	24.86
[10,]	0.074989	23.65	20.71	18.81	20.70
[11,]	0.100000	19.11	16.79	15.45	17.28
[12,]	0.133352	15.41	13.61	12.72	14.47
[13,]	0.177828	12.41	11.03	10.50	12.14
[14,]	0.237137	9.98	8.94	8.71	10.18
[15,]	0.316228	8.02	7.25	7.26	8.45
[16,]	0.421697	6.44	5.89	6.09	6.84
[17,]	0.562341	5.17	4.80	5.15	5.48
[18,]	0.749894	4.16	3.92	4.36	4.39
[19,]	1.000000	3.35	3.22	3.63	3.52