Statistical Process Control and R: A cautious beginning

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Statistical Process Control

Woodall (2000)

Statistical process control (SPC), a sub-area of SQC, consists of methods for understanding, monitoring, and improving process performance over time.

HERE:

We pick up the *monitoring* and get to control charting.

... or to change point detection, continuous inspection, surveillance, ...

The change-point model

Modeling of a stochastic process with a possible distributional change

Sequence of random variables $X_1, X_2, \ldots$ with pdf $\left\{F_{(i)}\right\}$ and a certain (unknown) time point $m =$ **change-point** with

$$F_{(i)} = \begin{cases} F_0 & , \ i < m \\ F_1 & , \ i \geq m \end{cases}.$$

Example: $F_0 = \mathcal{N}(\mu_0, 1)$, $F_1 = \mathcal{N}(\mu_1, 1)$ + independence

Notation:

$\left\{X_i\right\}_{i=1}^{m-1}$ – process **in control**,

$\left\{X_i\right\}_{i=m}^{\infty}$ – process **out of control**.

Control charts – SPC at work

Aim: Detect rapidly and reliably, whether there appeared change-point m!

- Transformation $\{X_i\}_{i=1,2,\ldots,n} \to Z_n$ and
- Stopping time $L = \min\{n \in \mathrm{N} : Z_n \notin \mathcal{O}\}$,

$$\mathcal{O} = (-\infty, ucl],\ [lcl, ucl],\ [lcl, \infty) \ldots$$

At time point L observation is stopped & the scheme signals an **alarm**.

L is a random value on $\mathrm{N} = \{1, 2, 3, \ldots\}$.

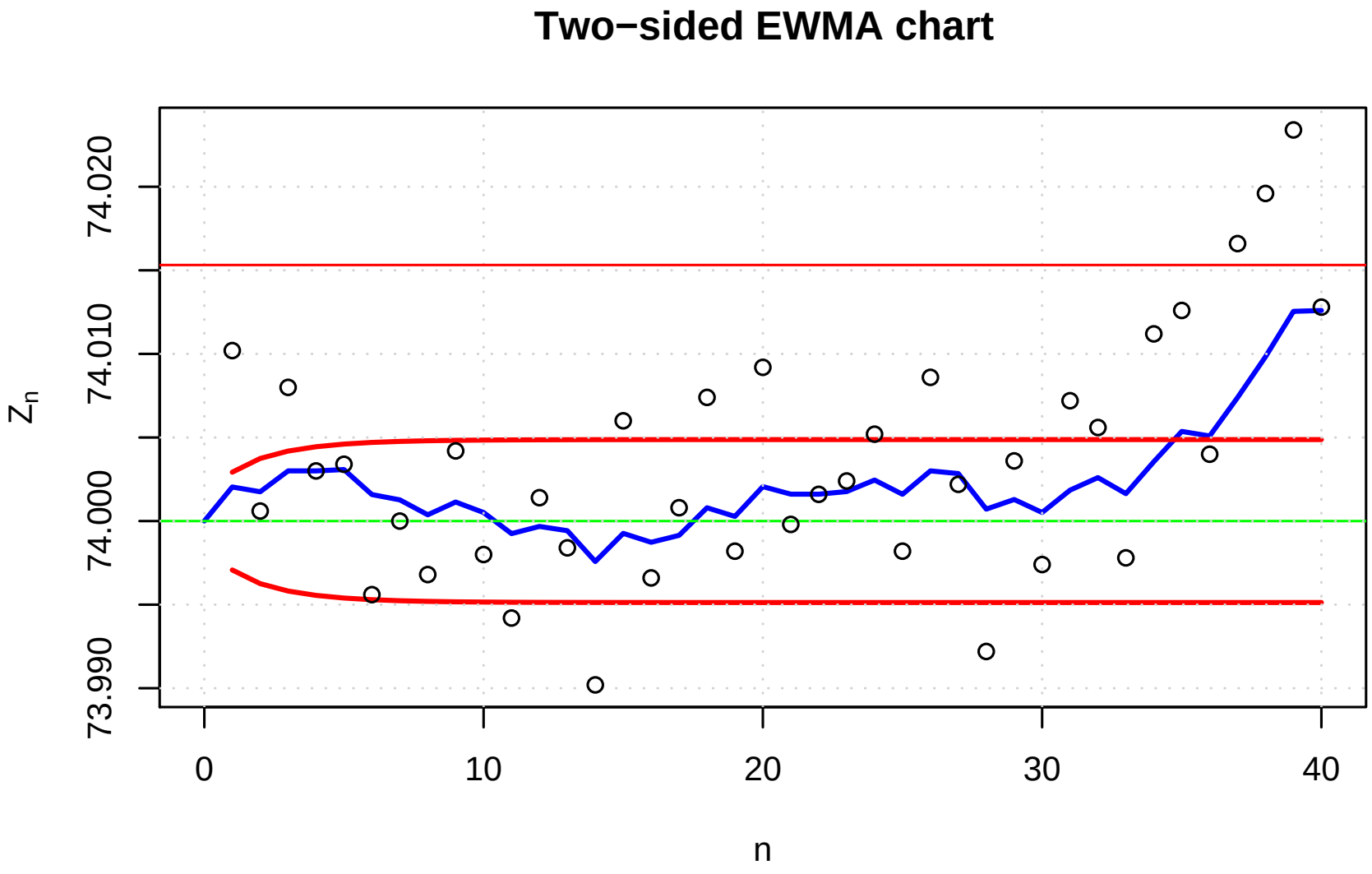
Control charts examples

- (One-sided) CUSUM: Page (1954)

$$Z_n = \max\left\{0, Z_{n-1} + X_n - k\right\}, \; Z_0 = z_0\,,$$
$$L = \inf\left\{n \in \mathsf{N} : Z_n > h\right\} \qquad \left(k = (\mu_0 + \mu_1)/2\right).$$

- (Two-sided) EWMA: Roberts (1959)

$$Z_n = (1-\lambda)Z_{n-1} + \lambda X_n\,, \; Z_0 = z_0\,,$$
$$L = \inf\left\{n \in \mathsf{N} : |Z_n - \mu_0| > c\sqrt{\lambda\left(1-(1-\lambda)^{2n}\right)/(2-\lambda)}\right\}.$$



Montgomery's piston ring diameters (taken from package qcc)

Popular performance measures

Notation: $E_m(.)$ expectation for given change-point m.

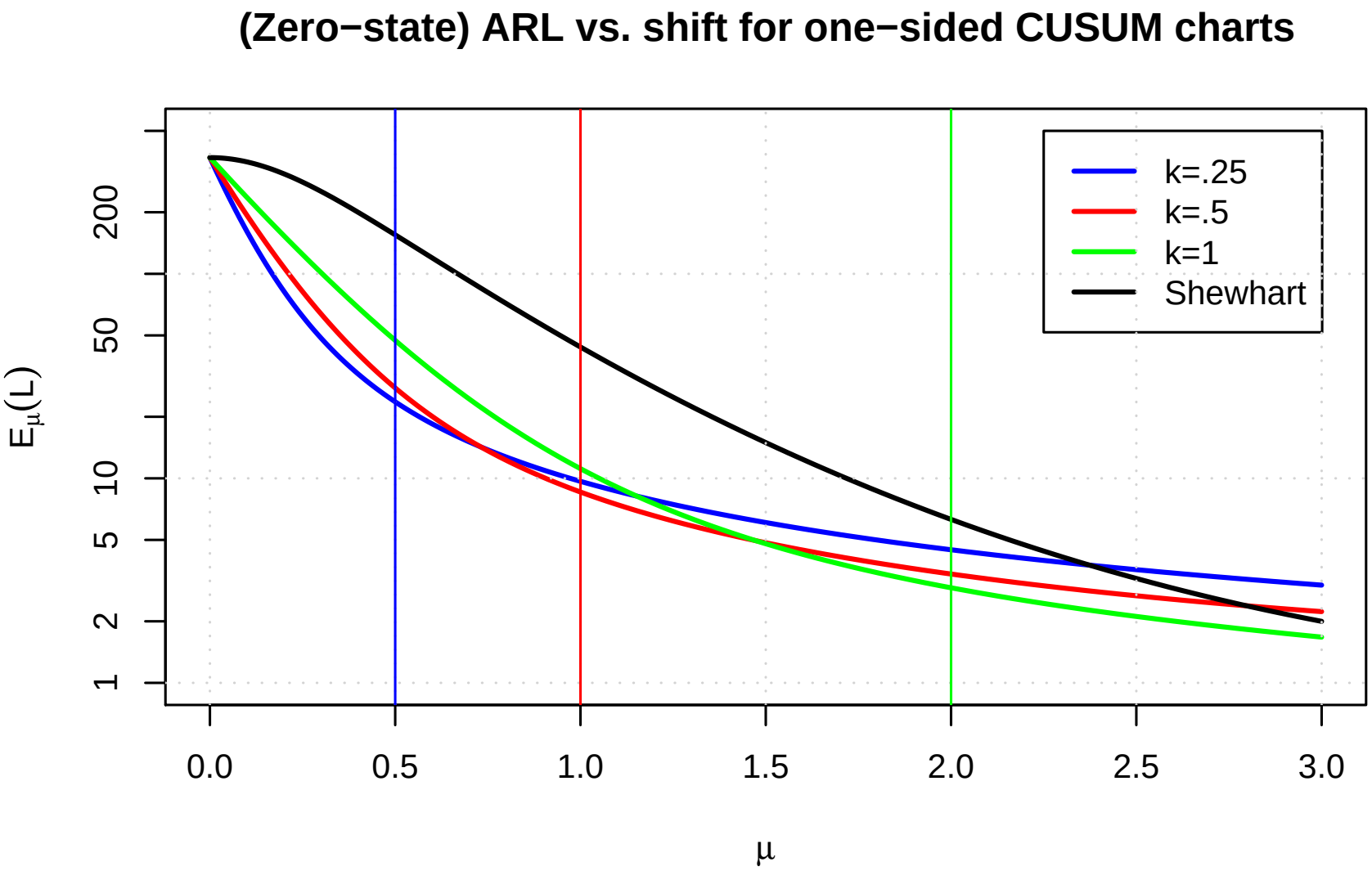
- Zero-State Average Run Length (ARL)

$$= \begin{cases} E_\infty(L) & \text{, process in control} \\ E_1(L) & \text{, process out of control} \end{cases}$$

- Steady-State ARL

$$= \lim_{m \to \infty} E_m\big(L - m + 1 \,|\, L \geq m\big)$$

- Design rules:
  - Determine alarm threshold in order to ensure $E_\infty(L) = A$ for given value A.
  - The remaining parameter is chosen to minimize one of the above ARL types for a certain out-of-control value.


(Zero−state) ARL vs. shift for one−sided CUSUM charts
$E_\mu(L)$
1
2
5
10
50
200
0.0
0.5
1.0
1.5
2.0
2.5
3.0
μ
k=.25
k=.5
k=1
Shewhart

R package spc provides

`xcusum.ad`	steady-state ARLs of CUSUM charts
`xcusum.arl`	(zero-state) ARLs of CUSUM charts
`xcusum.crit`	decision intervals of CUSUM charts
`xewma.ad`	steady-state ARLs of EWMA charts
`xewma.arl`	(zero-state) ARLs of EWMA charts
`xewma.crit`	critical values of EWMA charts

ARL computations – what do the others?

- The packages qcc of S-Plus 6 or R 1.9.0 provide facilities for drawing control charts, but no routines for choosing appropriate alarm thresholds.
- And so does SPSS 12 by means of SigmaPlot.
- SAS 8.2: Chart graphs and functions for computing the ARL. Less accurate for small λ in case of EWMA control charts.
- STATISTICA 6: It seems so that no ARL analyses are possible.
- MINITAB 14: The same seems to be valid.
- WinSPC: Only $\bar{X}$ and R charts are available.
- XploRe 4.6: XFG package spc – ARL computation based on a Markov chain approximation.

Things ought to be done next

- Write Manual to provide usability.
- Add ARL functions for variance control charts (based on S^2).
- Improve the plausibility check of the function arguments.
- Expose the package to practice.
- Look for "joint ventures" with packages qcc or strucchange or ...